Pure-Mode Network Analyzer for On-Wafer Measurements of Mixed-Mode *S*-Parameters of Differential Circuits

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***Abstract*— A practical measurement system is introduced for measurement of combined differential and common-mode (mixed-mode) scattering parameters, and its operation is discussed. A pure-mode system measures network parameters of a differential circuit in the fundamental modes of operation, and has improved accuracy over a traditional network analyzer for the measurement of such circuits. The system is suitable for on-wafer measurements of differential circuits. The transformation between standard s-parameters and mixed-mode s-parameters is developed. Example microwave differential structures are measured with the pure-mode vector-network analyzer (PMVNA), and the corrected data is presented. These structures are simulated, and the simulated mixed-mode s-parameters correlate well with the measured data.**

***Index Terms*—Differential analyzers, integrated circuit testing, measurement, multiport circuits, network testing.**

I. Introduction

As differential circuit applications become more common at RF and microwave frequencies, the need for accurate measurements of these circuits has become necessary. Scattering parameters (s-parameters) are well adapted to accurate measurements of linear networks at these frequencies. However, straightforward application of traditional s-parameter techniques does not lead to easily understandable characterization of a generic differential circuit.

Recent literature [1] has extended traditional theory so that general differential circuits can be easily analyzed with s-parameters. In general, a differential circuit responds to both a differential-mode and a common-mode stimulus. Therefore, a complete characterization of a differential circuit includes the differential-mode, common-mode, and any mode conversion responses. The representation of all these responses with s-parameters has been called mixed-mode s-parameters.

This paper first demonstrates the relationship between standard s-parameters and mixed-mode s-parameters. This relationship is considered as a basis for using a traditional four-port vector-network analyzer (VNA) for measurements of differential circuits, but it is found that a pure-mode measurement system, known as a pure-mode vector-network analyzer (PMVNA), provides more accurate measurements of differential circuits. This paper then introduces a pure-mode measurement system for mixed-mode s-parameters and measurements of example microwave differential structures.

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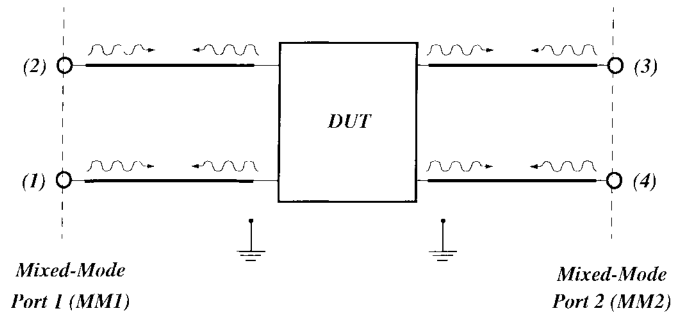


Fig. 1. Schematic of general two-port differential device.

This paper is organized as follows. In Section II, the transformation between standard s-parameters and mixed-mode s-parameters is developed. The PMVNA measurement system is presented in Section III. Section IV introduces test structures and their corrected measured data, and compares this with simulated mixed-mode s-parameters. Finally, conclusions are presented in Section V.

II. Transformation Between Standard and Mixed-Mode s-Parameters

To develop the transformation between standard s-parameters and mixed-mode s-parameters, the mixed-mode s-parameters must first be defined. Consider the differential circuit with two differential ports as shown in Fig. 1. In general, each port can support the propagation of differential-mode and common-mode waves [2]. The response of the differential circuit to a stimulus can be expressed with s-parameters in terms of these modes [1] by

$$\begin{bmatrix} b_{d1} \\ b_{d2} \\ b_{c1} \\ b_{c2} \end{bmatrix} = \left[\begin{array}{c|c} S_{dd} & S_{dc} \\ \hline S_{cd} & S_{cc} \end{array}\right] \begin{bmatrix} a_{d1} \\ a_{d2} \\ a_{c1} \\ a_{c2} \end{bmatrix} \tag{1}$$

where each partition represents a two-by-two s-parameter submatrix. The partition labeled S_{dd} are the differential s-parameters, S_{cc} are the common-mode s-parameters, and S_{dc}

and S_{cd} are the mode-conversion or cross-mode s-parameters, where S_{dc} describes the conversion of common-mode waves into differential-mode waves, and S_{cd} describes the conversion of differential-mode waves into common-mode waves. Together, these four sets of s-parameters simultaneously describe the combined differential and common-mode behavior of the test device, and are, hence, known as mixed-mode s-parameters. In (1), a_{di} and b_{di} are the normalized differential-mode stimulus and response waves, and a_{ci} and b_{ci} are the normalized common-mode stimulus and response waves. The definitions of these normalized waves for each mode, developed in [1], are repeated here as follows:

$$\begin{aligned}
a_{di} &= \frac{1}{2\sqrt{R_d}}\left[v_{di} + i_{di}R_d\right]\\
a_{ci} &= \frac{1}{2\sqrt{R_c}}\left[v_{ci} + i_{ci}R_c\right]\\
b_{di} &= \frac{1}{2\sqrt{R_d}}\left[v_{di} - i_{di}r_d\right]\\
b_{ci} &= \frac{1}{2\sqrt{R_c}}\left[v_{ci} - i_{ci}R_c\right]
\end{aligned} \tag{2}$$

where the subscript i indicates a port number. Here, v_{di} and i_{di} represent the differential-mode voltage and current at port i, and v_{ci} and i_{ci} are the common-mode voltage and current at the same port. R_d and R_c represent the differential and common-mode characteristic (or reference) resistances, respectively.

The mixed-mode s-parameters in (1) can be directly related to standard four-port s-parameters by examining the relations between mixed-mode normalized waves in (2) and traditional normalized waves. If nodes 1 and 2 are paired as a single differential port, and nodes 3 and 4 are also paired, it is shown in [1]

$$\begin{aligned}
a_{d1} &= \frac{1}{\sqrt{2}}(a_1 - a_2)\\
a_{c1} &= \frac{1}{\sqrt{2}}(a_1 + a_2)\\
b_{d1} &= \frac{1}{\sqrt{2}}(b_1 - b_2)\\
b_{c1} &= \frac{1}{\sqrt{2}}(b_1 + b_2)
\end{aligned} \tag{3}$$

$$\begin{aligned}
a_{d2} &= \frac{1}{\sqrt{2}}(a_3 - a_4)\\
a_{c2} &= \frac{1}{\sqrt{2}}(a_3 + a_4)\\
b_{d2} &= \frac{1}{\sqrt{2}}(b_3 - b_4)\\
b_{c2} &= \frac{1}{\sqrt{2}}(b_3 + b_4)
\end{aligned} \tag{4}$$

where a_i and b_i are the waves measured at ports 1–4.

By using the definition of s-parameters [3] for a four-port network together with the relations in (3) and (4), a transformation between mixed-mode and standard s-parameters can be found. The transformation can be developed by considering the relationships between the standard and mixed-mode incident waves, a, which can be written as

$$\begin{bmatrix} a_{d1}\\ a_{d2}\\ a_{c1}\\ a_{c2}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 & 0 & 0\\ 0 & 0 & 1 & -1\\ 1 & 1 & 0 & 0\\ 0 & 0 & 1 & 1\end{bmatrix}\begin{bmatrix} a_1\\ a_2\\ a_3\\ a_4\end{bmatrix} \tag{5}$$

or $A_{mm} = MA_{\mathrm{std}}$, where A_{mm} and A_{std} are the mixed-mode a-waves, respectively, and

$$M = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 & 0 & 0\\ 0 & 0 & 1 & -1\\ 1 & 1 & 0 & 0\\ 0 & 0 & 1 & 1\end{bmatrix}. \tag{6}$$

Similarly, for the response waves, b, it is found $B_{mm} = MB_{\mathrm{std}}$. Applying the generalized definition of s-parameters $B = SA$ [3], [4], it can be shown that

$$S_{mm} = MS_{\mathrm{std}}M^{-1} \tag{7}$$

where S_{mm} are the mixed-mode s-parameters and S_{std} are the standard four-port s-parameters. Additionally, M has the property $M^{-1} = M^T$.

The transformation in (7) gives additional insight into the nature of mixed-mode s-parameters. The transformation is a similarity transformation, which indicates that a change of basis has occurred between standard and mixed-mode s-parameters. Further, it clearly indicates that the two sets of s-parameters are different representations of the same device, and that ideally, the two representations contain the same information about the device.

The existence of a transformation between standard and mixed-mode s-parameters suggests two possible approaches to the measurement of differential circuits. One approach is the use of a traditional four-port VNA. A traditional VNA would measure standard s-parameters by stimulating each terminal of the differential circuit individually, and these s-parameters would then be transformed to mixed-mode s-parameters for analysis. Alternatively, the mixed-mode s-parameters of the differential circuit can be measured directly by stimulating each mode individually. A network analyzer which directly measures mixed-mode s-parameters will be referred to as a PMVNA.

However, the two approaches do not yield equally accurate mixed-mode s-parameters of differential devices. It has been shown that the PMVNA has an accuracy advantage over a traditional four-port VNA while measuring a differential circuit [5]. Mixed-mode s-parameters generated by transforming standard s-parameters measured by a traditional four-port VNA exhibit higher levels of uncertainty than those measured by a PMVNA. In particular, the uncertainties of transformed mode-conversion parameters, S_{dc} and S_{cd}, can be significantly larger than the actual parameter magnitudes. Although it is beyond the scope of this paper to present the accuracy comparison of the two approaches, the accuracy advantage of a pure-mode measurement system provides motivation for the development of a specialized measurement system for differential circuits.

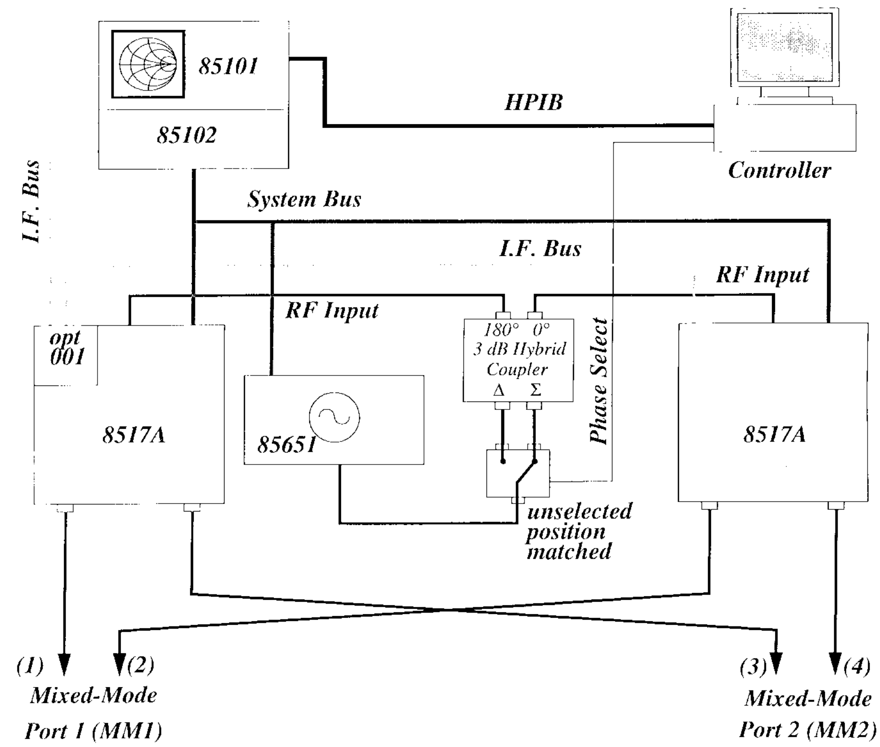


Fig. 2. Block diagram of PMVNA as implemented.

III. Measurement System Implementation and Operation

Since there are measurement accuracy advantages, the goal of a PMVNA is to measure the response of a differential device-under-test (DUT) to its natural input modes, namely differential mode and common mode. A PMVNA stimulates the DUT with each mode individually, and each response is decomposed into its constituent modes. For purposes of s-parameter calculation, the stimulus will be described in terms of a normalized incident differential or common-mode wave, a_c or a_d, and the response in terms of a normalized transmitted or reflected differential or common-mode wave, b_c or b_d. The various ratio's of a's and b's will be the s-parameters, which are the quantities of interest.

The PMVNA, as implemented for this paper, is based on the 8510C-modular-network-analyzer system manufactured by Hewlett-Packard. A block diagram of the measurement system is given in Fig. 2. The PMVNA system employs two 8517A test sets, both of which have had minor control hardware modifications to allow the simultaneous operation of both test sets. Ports 1 and 2 form mixed-mode port 1 (denoted as MM1), and ports 3 and 4 are paired to form mixed-mode port 2 (denoted as MM2). The RF inputs to the two test sets are derived from a single 85 651 RF source via a 0°/180° 3-dB hybrid splitter. An RF switch allows selection of the phase difference between the inputs to the test sets. Included in one of the test sets is option 001, which multiplexes the IF signals of the two test sets, allowing the use of a single 85 102 IF detector and an 85 101 display/processor. The 85 102 IF detector down-mixes the signals to a second IF and digitizes them, and the 85 101 display/processor provides local control of the system components through the system bus [6]. The frequencies of operation of the PMVNA shown in Fig. 2 are set by the hybrid coupler, and the coupler used is specified to operate from 1.0 to 12.4 GHz.[1] The overall control of the PMVNA is provided by software on a computer controller, and the software is constructed in LabVIEW,[2] executed on a Sun Sparc2.

Careful attention must be given to the signal launch from the probe tip to the wafer surface. As shown in [1], the mixed-mode s-parameters of an arbitrary differential DUT can be accurately measured with uncoupled reference transmission lines (or ports), independent of any coupled modes of propagation that may exist in the DUT. This is achieved through the decomposition of any coupled-mode signals into uncoupled modes, which results in mixed-mode s-parameters that are normalized to the reference impedance of the uncoupled lines. Accordingly, the wafer probes that interface with a differential DUT can be composed of isolated single-ended probes.

In order to maintain a smooth transition to any coupled-modes, two single-ended probes are paired into a single mixed-mode probe. Each mixed-mode probe provides two RF-measurement ports which are in reasonably close proximity, but are ideally uncoupled. Hence, a mixed-mode probe footprint of GS_1GS_2G is adopted. The PMVNA system, as implemented for this paper, is fitted with a pair of 150-μm-pitch dual-RF probes manufactured by GGB Industries.[3]

[1] *Product Data Pack: HJM-4R-6.5G 180° Power Divider/Combiner,* Merrimac Industries, West Caldwell, NJ, June 1993.

[2] *LabVIEW: User Documentation,* National Instruments, Austin, TX, Aug. 1993.

[3] *Product Literature: Dual Model 40A Probes,* GGB Industries, Naples, FL, 1996.

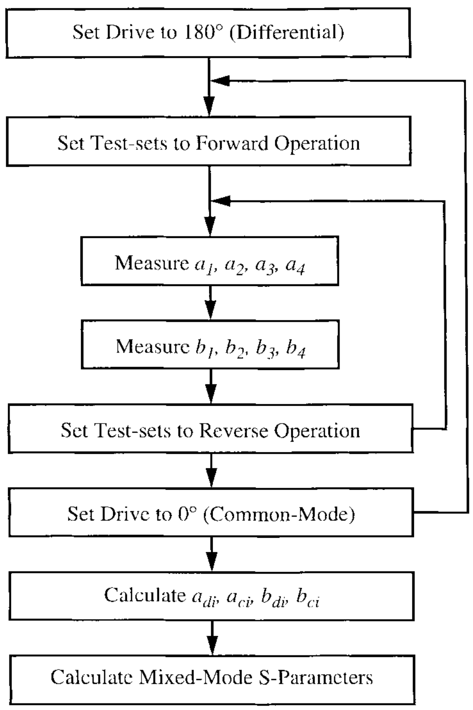


Fig. 3. Flow chart of the operation of the PMVNA.

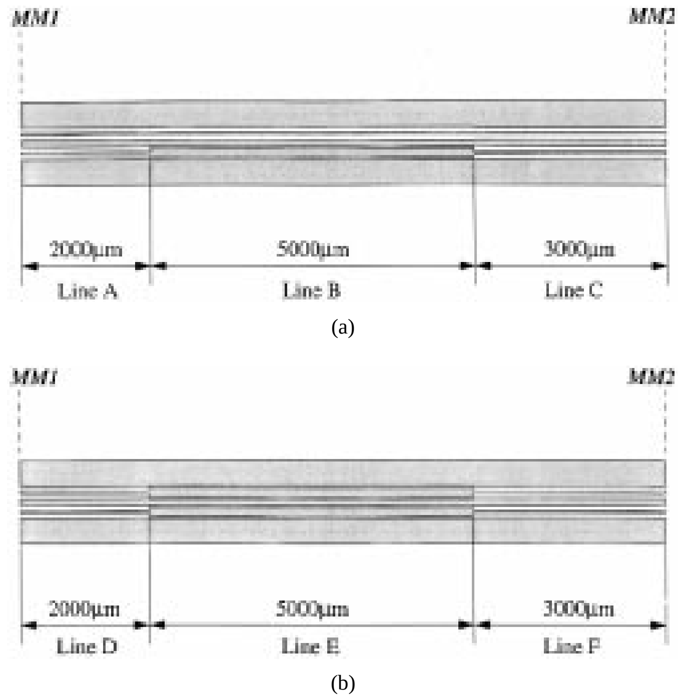


Fig. 4. Layout of the measured differential structures. (a) Transmission-line pair with an asymmetric step in characteristic impedance. (b) Transmission-line pair with a symmetric step in characteristic impedance.

The basic operation of the PMVNA implements both a differential and a common-mode drive for the DUT. Referring to the flow diagram in Fig. 3, the PMVNA first measures the DUT with a differential stimulus. Forward operation of the DUT is measured by driving nodes 1 and 2 with a nominal 180° phase difference. The incident, transmitted, and reflected normalized waves are measured at node pair 1 and 2, and pair 3 and 4, respectively. Next, reverse operation of the DUT is measured by driving nodes 3 and 4, again with 180° phase difference, and the normalized waves are measured at all nodes. The DUT is then measured with a common-mode stimulus. Forward operation is measured by driving nodes 1 and 2 with a nominal 0° phase difference. Again, the incident, transmitted, and reflected waves are measured. Finally, the common-mode reverse stimulus is generated, and all the normalized waves are again measured. From these measurements, all mixed-mode s-parameters can be calculated by

$$\begin{aligned} S_{ddij} &= \left.\frac{b_{di}}{a_{dj}}\right|_{a_{ci}=a_{cj}=a_{dj}=0} \\ S_{dcij} &= \left.\frac{b_{di}}{a_{cj}}\right|_{a_{di}=a_{cj}=a_{dj}=0} \\ S_{cdij} &= \left.\frac{b_{ci}}{a_{dj}}\right|_{a_{ci}=a_{cj}=a_{dj}=0} \\ S_{ccij} &= \left.\frac{b_{ci}}{a_{cj}}\right|_{a_{di}=a_{cj}=a_{dj}=0} \end{aligned} \tag{8}$$

with $i, j = \{1, 2\}$.

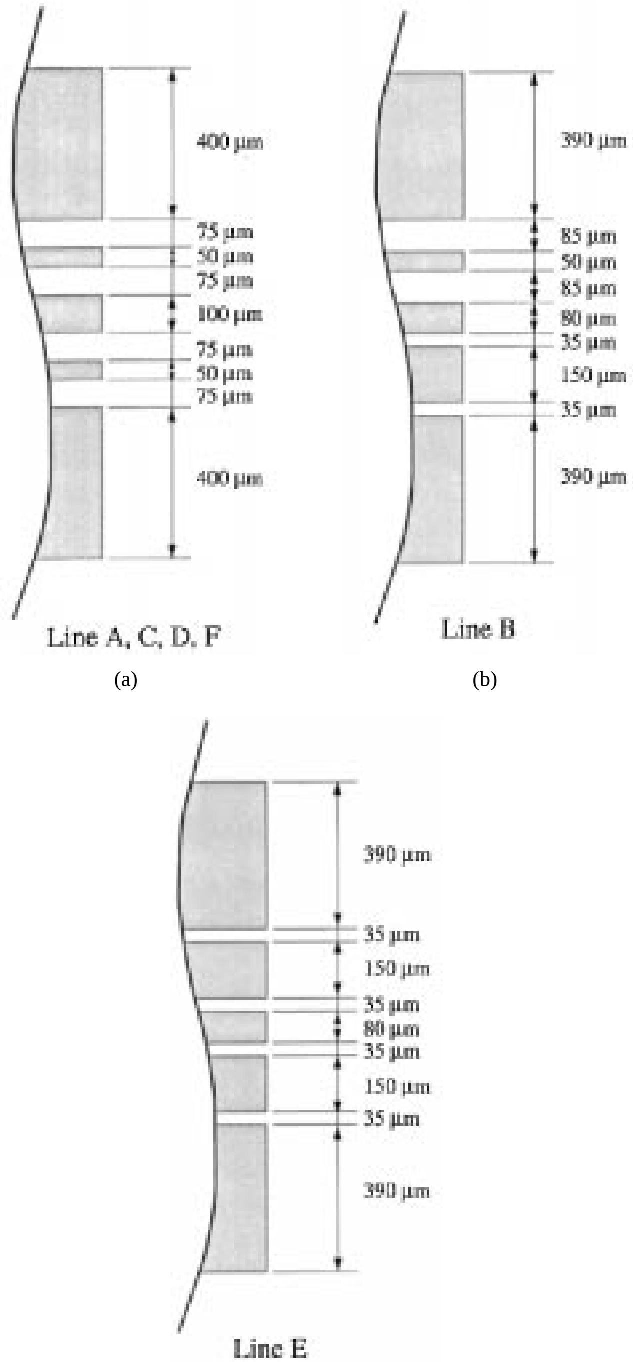
The calibration of the PMVNA is critical for accurate RF and microwave measurements. The discussion of the details of this aspect of the PMVNA operation is beyond the scope of this paper, but will be presented in detail in a future paper.

A few summary comments about the calibration are appropriate. First, the calibration as implemented can correct for phase and magnitude imbalance in the differential/common-mode signal generation. This imbalance is generated by imbalances in the hybrid coupler and by unbalanced RF signal paths in the two test sets. Any imbalance in the PMVNA causes an unintended mode to be stimulated, as shown in [7]. Proper calibration can correct for a reasonable amount of imbalance, although the unintended signals are still generated. Through calibration, the useful frequency of operation of the PMVNA (again limited by the hybrid coupler) is extended from 0.1 to over 20 GHz.

A second issue of calibration is port isolation correction. The ideal PMVNA system would maintain a high level of isolation between all four measurement ports up to the measurement reference plane. While easy to do with coaxial cable, this isolation is more difficult with the planar structures of a wafer-probe system. However, the coupling between the ports is relatively weak, and can be neglected in most cases. In this paper, there is no isolation correction, so the port coupling is neglected, although it remains.

IV. Measurement Examples

To illustrate the measurement capabilities of the PMVNA, two simple differential structures have been measured and compared to simulated results. These structures have been chosen because they provide good examples of mixed-mode behavior, such as mode conversion. Furthermore, these structures can be simulated with a reasonably high degree of confidence in the accuracy of the results.



(a) (b) (c)

Fig. 5. Detail of layouts showing dimensions of (a) transmission-line pair A, C, D, F, (b) transmission-line pair B, and (c) transmission-line pair E.

The devices, shown in Fig. 4, are stepped-impedance transmission-line pairs. Both are fabricated with thin-film gold (4-μm thick $\pm$0.25 mm) on a polished alumina substrate ($\varepsilon_r \approx 9.9$, $\tan\delta \approx 0.001$). The first structure, shown in Fig. 4(a), has an asymmetric step in the impedances of the transmission-line pair. It has three sections of different impedances, where Lines A and C are pairs of nominally 69-Ω transmission lines, and Line B provides a pair of transmission lines with different impedances (one 72 Ω, the other 40 Ω). The lengths of the three sections are shown in Fig. 4(a), and are known ± 1 μm. The second structure, shown in Fig. 4(b), is a symmetrically stepped-impedance transmission-line pair. Lines D and F are also nominally 69-Ω transmission lines, and Line E provides a pair of transmission lines with equal impedances (40 Ω). The vertical dimensions of the two structures are shown in Fig. 5. The tolerance on the spaces between the conductors is ± 1 μ m.

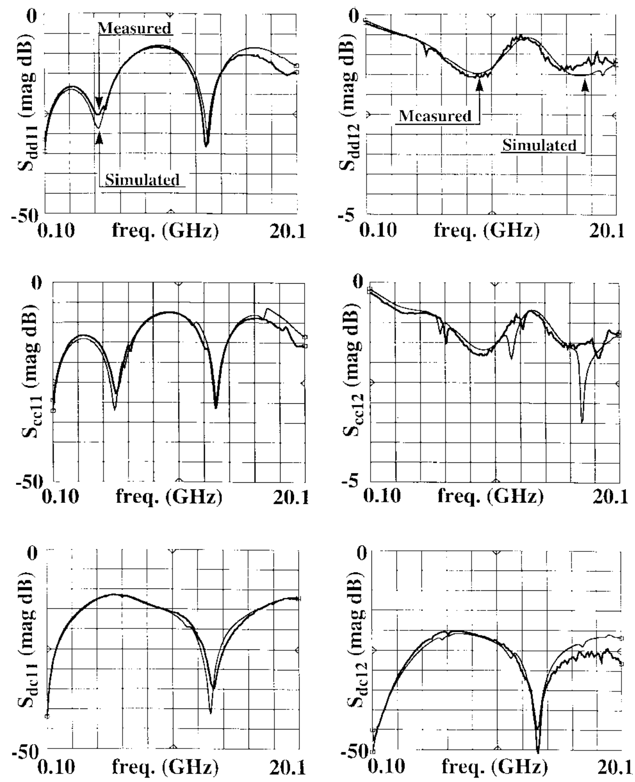


Fig. 6. Selected measured and simulated mixed-mode s-parameters of the asymmetric-stepped transmission-line pair. The heavy line indicates measured data, while the lighter line indicates simulated data.

The two structures have been simulated in Hewlett-Packard's microwave design system (MDS).[4] The simulation employs the five conductor-coupled microstrip-transmission-line model incorporated within MDS with the first, third, and fifth conductor grounded. The transitions between the sections of line are modeled with the MDS microstrip step-in-width model, but the coupling between each step is not modeled. The simulation makes a quasi-static approximation, but includes metal losses ($\sigma = 4.1 \times 10^7$ S/m) and dielectric losses. The nominal values for all dimensions and physical properties are used in all simulations, with the exception of the end sections (Lines A, C, D, F). The lengths of these sections are reduced by the distance the probes overlap the end of the structures. It is assumed that the overlap is 25 μm at each end, but the actual overlap can vary, causing discrepancies between measured and simulated data. The simulations generate standard s-parameters, which are converted to mixed-mode s-parameters through the transformation (7).

The corrected measured and simulated mixed-mode s-parameters of the asymmetrically stepped transmission-

[4] *HP 85150B Microwave and RF Design Systems: User Documentation,* Hewlett-Packard, Santa Rosa, CA, 1992.

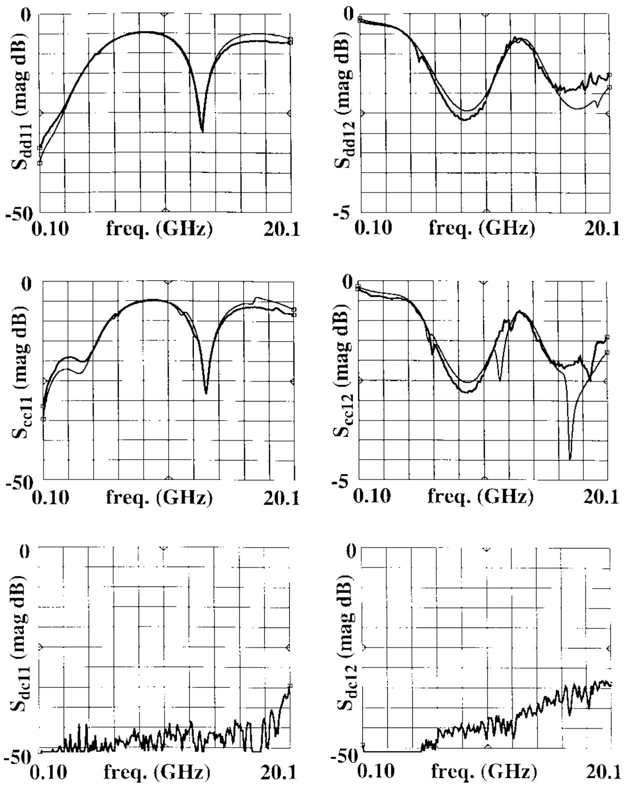


Fig. 7. Selected measured and simulated mixed-mode s-parameters of the symmetric-stepped transmission-line pair. The heavy line indicates measured data, while the lighter line indicates simulated data.

line pair are plotted in Fig. 6. The agreement between the measured and simulated data is quite good across the entire bandwidth. The measured and simulated data share many of the unusual fine features of the responses. For example, the measured S_{cc12} has several abrupt increases in insertion loss at about 6.1, 12.2, and 18.3 GHz; likewise, the simulated data show similar responses, although at slightly lower frequencies. Additionally, the device demonstrates a strong level of mode conversion (S_{dc}, S_{cd}), and the agreement between measured and simulated mode conversion indicates that the effects of imbalance in the PMVNA are largely removed through calibration.

The results of the first test structure illustrate the advantage of the mixed-mode concepts. The mixed-mode s-parameter concepts allow simple treatment of asymmetrical devices. As shown in [8], a pair of coupled lines which is asymmetric (e.g., the two lines have different dimensions) does not support differential and common-mode propagation, but rather it supports the π and c modes, of which differential and common modes are special cases. However, mixed-mode s-parameters provide an equivalent representation of an asymmetric device in terms of the effective differential, common-mode, and mode-conversion responses. Referring again to Fig. 6, the measured pure differential parameters (S_{dd}), common-mode parameters (S_{cc}), and mode-conversion parameters (S_{dc}), all show approximately periodic variations across frequency. These variations are analogous to the effects of a single transmission line with a step-in impedance. Hence, each partition of the mixed-mode s-parameter matrix (S_{dd}, S_{cc}, S_{cd}, S_{dc}) can be interpreted as an effective single transmission line with stepped impedance.

The corrected measured and simulated mixed-mode s-parameters of the symmetrically stepped transmission-line pair are plotted in Fig. 7. Again, the agreement between measured and simulated data is good. Like the asymmetric structure, the symmetric structure demonstrates the periodic responses analogous to a single transmission line with a step in impedance. The symmetric structure only supports differential and common mode, and hence, no significant mode-conversion occurs. The measured mode-conversion s-parameters are small, and are most likely generated by probe crosstalk and residual errors in the calibration.

In the interest of space, only six of the mixed-mode s-parameters are included for each of the test structures, yet agreement of the remaining parameters is very similar. Since the measured device is reciprocal, there are ten unique mixed-mode s-parameters. The mixed-mode s-parameter matrix of a reciprocal device is symmetric along the diagonal. The devices measured did not possess port symmetry, but in such a case, there would be only six unique mixed-mode s-parameters.

V. Conclusions

A practical PMVNA system has been described and implemented. The system is constructed from readily available test equipment. Some simple differential structures have demonstrated the usefulness of the concept of mixed-mode s-parameters and the good accuracy of the measurement system compared to simulations. A transformation has been developed between standard s-parameters and mixed-mode s-parameters, which provides a simple method by which to calculate one set of parameters from the other. The primary advantage of the transformation is the ability to generate simulated mixed-mode s-parameters from computer-aided design (CAD) tools, which provide standard s-parameters. Although the transformation could ideally be used to allow a traditional VNA to make measurements of mixed-mode s-parameters, a pure-mode system is necessary to accurately measure the mode-conversion in real integrated differential test structures.

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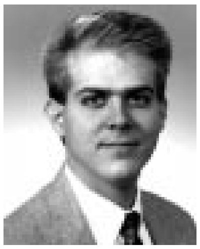

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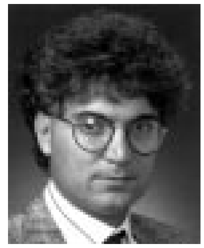

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