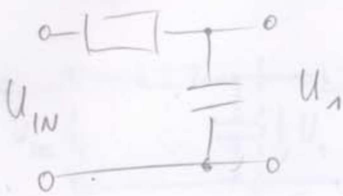
1. Ordnung



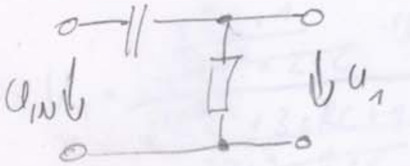
$$U_{IN} = R + \frac{1}{sC}$$

$$U_1 = \frac{1}{sC}$$

$$G(s) = \frac{U_1}{U_{IN}} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{\cancel{\frac{1}{sC}}}{\frac{sRC+1}{\cancel{sC}}}$$

$$G(s) = \frac{1}{sRC+1}$$

Nur Nebenbei: Vielleicht kann man das ganze auch mal für einen Hochpass durchspielen.



$$U_{IN} = \frac{1}{sC} + R$$

$$U_1 = R$$

$$G(s) = \frac{U_1}{U_{IN}} = \frac{R}{R + \frac{1}{sC}} = \frac{1}{1 + \frac{1}{sRC}}$$

⋮

2. Ordnung

3. —"—

4. —"—

⋮

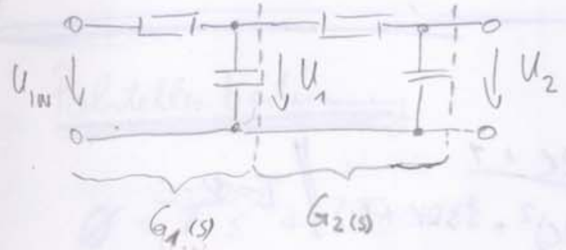
2.Ordnung

$$U_{IN} = R + \cfrac{1}{sC + \cfrac{1}{R + \cfrac{1}{sC}}} \qquad U_2 = \frac{1}{sC}$$

$$U_1 = \cfrac{1}{sC + \cfrac{1}{R + \cfrac{1}{sC}}}$$

$U_1 = U_{IN}$ aus 1. Ordnung!

$$U_1 = R + \frac{1}{sC}$$



$$G_1(s) = \frac{U_1}{U_{IN}} = \frac{\cfrac{1}{sC + \cfrac{1}{R + \cfrac{1}{sC}}}}{R + \cfrac{1}{sC + \cfrac{1}{R + \cfrac{1}{sC}}}} \Rightarrow U_1 = \frac{U_{IN} \cdot \cfrac{1}{sC + \cfrac{1}{\frac{sRC+1}{sC}}}}{R + \cfrac{1}{sC + \cfrac{sC}{sRC+1}}}$$

mit TI (A)
↳

$$U_1 = \frac{\cfrac{1}{\frac{sC(sRC+1)+sC}{sRC+1}} \cdot U_{IN}}{R + \cfrac{1}{\frac{sC(sRC+1)+sC}{sRC+1}}} = \frac{\cfrac{sRC+1}{s^2RC^2 + 2sC} \cdot U_{IN}}{R + \cfrac{sRC+1}{s^2RC^2 + 2sC}}$$

$$U_1 = \frac{\cfrac{sRC+1}{\cancel{s^2RC^2+2sC}} \cdot U_{IN}}{\cfrac{s^2R^2C^2 + 3sRC + 1}{\cancel{s^2RC^3 + 2sC}}} \qquad U_1 = \frac{sRC+1 \quad \cdot U_{IN}}{(sRC)^2 + 3sRC + 1}$$

↓ vom U_1 nach U_2

$$\frac{U_2}{U_1} = G_2(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} \quad \Big\| \; U_1 \text{ ermitteln}$$

↓ von U_2 nach

$$\Rightarrow U_2 = \frac{U_1 \cdot \frac{1}{sC}}{R + \frac{1}{sC}} = \frac{\cfrac{U_{IN} \cdot (sRC+1)}{(sRC)^2 + 3sRC + 1} \cdot \cfrac{1}{sC}}{R + \cfrac{1}{sC}} \quad \Big| : U_{IN}$$

$$\frac{U_2}{U_{IN}} = G(s) = \frac{\cfrac{sRC+1}{s^3R^2C^3 + 3s^2RC^2 + sC}}{\cfrac{sRC+1}{sC}} = \frac{sC}{s^3R^2C^3 + 3s^2RC^2 + sC} \Rightarrow \boxed{G(s) = \frac{1}{(sT)^2 + 3sT + 1}\Bigg|_{T=R\cdot C}}$$

(*) ↳

(2)

Ⓐ Ergäbe in T 1

$$\frac{U_1}{U_{IN}} = \frac{sRC+1}{(sRC)^2+3sRC+1}$$

$$\Rightarrow G(s) = \underbrace{G_1(s)}_{\text{aus 1. Ordnung}} \cdot U_1 = \frac{1}{sRC+1} \cdot \frac{sRC+1}{(sRC)^2+3sRC+1} \quad \Big\| \; T = R \cdot C$$

$$\boxed{G(s) = \frac{1}{(sT)^2+3sT+1}}$$

Ⓑ $G(s) = \underbrace{G(s)}_{\text{aus 1. Ordnung}} \cdot U_1 = \frac{1}{\cancel{sRC+1}} \cdot \frac{\cancel{sRC+1}}{(sRC)^2+3sRC+1}$

$$G(s) = \frac{1}{(sRC)^2+3sRC+1}$$

$$\Rightarrow G(s) = \boxed{\frac{1}{(sT)^2 + 3sT + 1}\Bigg|_{T = R\cdot C}}$$

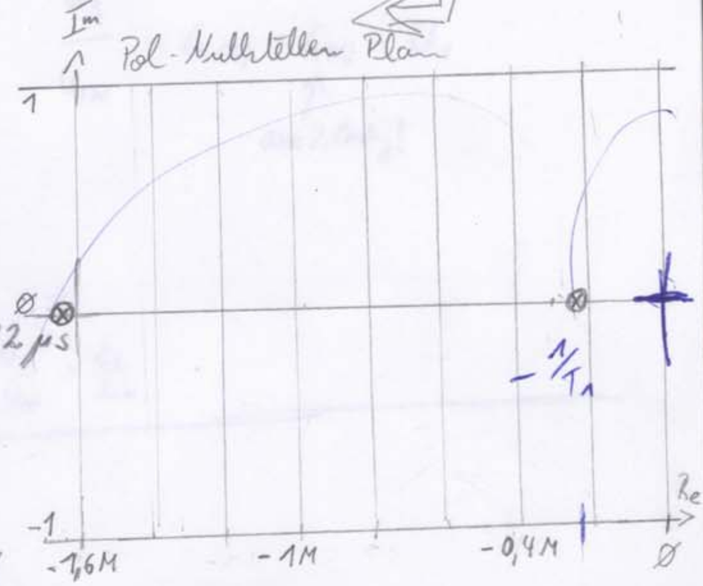
Polstellenbestimmung

$$\emptyset = T^2 s^2 + 3Ts + 1$$

$\xrightarrow{TI}$ $p_1 = \frac{-0{,}382}{T}$ $\quad p_2 = \frac{-2{,}618}{T}$

allg. Form
$$\frac{1}{T^n (s+p_1)(s+p_2)\cdots(s+p_n)}$$

$$G(s) = \frac{1/T^2}{\left(s + \frac{0{,}382}{T}\right)\left(s + \frac{2{,}618}{T}\right)}$$



laut Praktikum $T = 2\pi RC = 10\,\mu s$

$$\Longrightarrow R\cdot C = \frac{10\,\mu s}{2\pi} = 1{,}592\,\mu s$$

$$\longrightarrow p_1 = \frac{-0{,}382}{1{,}592\,\mu s} \qquad p_2 = \frac{-2{,}618}{1{,}592\,\mu s}$$

$$p_1 = -240\cdot 10^3\ Hz$$

$$p_2 = -1{,}645\cdot 10^6\ Hz$$

Amplitudengang für $n=2$

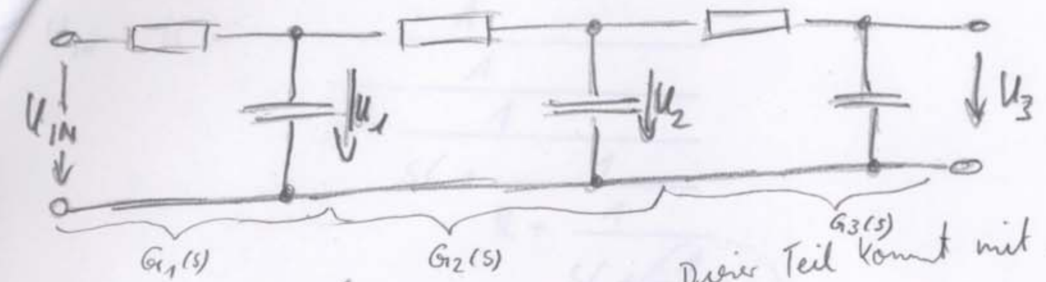
$$\frac{1}{\sqrt{2}} = \left|\frac{1}{(Ts)^2 + 3Ts + 1}\right| \overset{s=\omega}{=} \frac{1}{\sqrt{((T\omega)^2+1)^2 + 9(T\omega)^2}}$$

$\xrightarrow{T}$

Grenzfrequenz wie in Prak. Dig. Filter aus Amplitudengang ermitteln mit MatLab Bode ? z.B.

zu 2

3.Ordnung



$$U_{IN} = R + \frac{1}{sC + \frac{1}{R + \frac{1}{sC + \frac{1}{R + \frac{1}{sC}}}}}$$

Dieser Teil kommt mit jeder Erhöhung der Ordnungszahl erneut hinzu !

$$\frac{U_3}{U_{IN}} = G(s) = G(s) \cdot U_1$$

aus 2.Ordnung!

$$U_1 = \frac{1}{sC + \frac{1}{R + \frac{1}{sC + \frac{1}{R + \frac{1}{sC}}}}}$$

$$\frac{U_1}{U_{IN}} = \frac{Z_1}{Z_{in}}$$

$$U_1 = \frac{U_{IN} \dfrac{1}{sC + \frac{1}{R + \frac{1}{sC + \frac{1}{R + \frac{1}{sC}}}}}}{R + \dfrac{1}{sC + \frac{1}{R + \frac{1}{sC + \frac{1}{R + \frac{1}{sC}}}}}} = \frac{(sRC)^2 + 3sRC + 1}{(sRC)^3 + 5(sRC)^2 + 6sRC + 1}$$

Eingabe in TI

$$\frac{U_3}{U_{IN}} = G(s) \cdot U_1 \Big|_{\text{mit } T = R \cdot C} = \frac{1}{\cancel{(sT)^2 + 3sT + 1}} \cdot \frac{\cancel{(sT)^2 + 3sT + 1}}{(sT)^3 + 5(sT)^2 + 6sT + 1}$$

aus 2.Ordnung

$$\boxed{G(s) = \frac{1}{(sT)^3 + 5(sT)^2 + 6sT + 1}}$$

4.Ordnung

$$U_{IN} = R + \cfrac{1}{sC + \cfrac{1}{R + \cfrac{1}{sC + \cfrac{1}{R + \cfrac{1}{sC + \cfrac{1}{R + \frac{1}{sC}}}}}}}$$

$$U_1 = \cfrac{1}{sC + \cfrac{1}{R + \cfrac{1}{sC + \cfrac{1}{R + \cfrac{1}{sC + \cfrac{1}{R + \frac{1}{sC}}}}}}}$$

$$U_1 = \frac{U_{IN} \cdot Z_1}{Z_{IN}}$$ Eingabe in TI

$$\Rightarrow U_1 = \frac{(sRC)^3 + 5(sRC)^2 + 6\,sRC + 1}{(sRC)^4 + 7(sRC)^3 + 15(sRC)^2 + 10\,sRC + 1}$$

$$\boxed{G(s) = G(s) \cdot U_1 \Big|_{T=R\cdot C} = \frac{1}{(sT)^4 + 7\,(sT)^3 + 15\,(sT)^2 + 10\,sT + 1}}$$

aus 3. Ordnung

4

5.Ordnung

$$U_1 = \frac{U_{IN} \cdot Z_1}{Z_{IN}}$$ Eingabe in TI

$$U_1 = \frac{(sRC)^4 + 7(sRC)^3 + 15(sRC)^2 + 10\,sRC + 1}{(sRC)^5 + 9(sRC)^4 + 28(sRC)^3 + 35(sRC)^2 + 15\,sRC + 1}$$

$$\boxed{G(s) = G(s) \cdot U_1 \Big|_{T=R\cdot C} = \frac{1}{(sT)^5 + 9(sT)^4 + 28(sT)^3 + 35(sT)^2 + 15sT + 1}}$$

aus 4. Ordnung

6. Ordnung

$$U_1 = \frac{U_{IN} \cdot Z_1}{Z_{IN}}$$ Eingabe in TI

$$\rightarrow U_1 = \frac{(sRC)^5 + 9(sRC)^4 + 28(sRC)^3 + 35(sRC)^2 + 15\,sRC + 1}{(sRC)^6 + 11(sRC)^5 + 45(sRC)^4 + 84(sRC)^3 + 70(sRC)^2 + 21\,sRC + 1}$$

$$\Rightarrow \boxed{G(s) = \frac{1}{(sT)^6 + 11(sT)^5 + 45(sT)^4 + 84(sT)^3 + 70(sT)^2 + 21sT + 1}}$$

7. Ordnung

$$G(s) = \frac{1}{(sT)^7 + 13(sT)^6 + 66(sT)^5 + 165(sT)^4 + 210(sT)^3 + 126(sT)^2 + 28sT + 1}$$

8. Ordnung

$$G(s) = \frac{1}{(sT)^8 + 15(sT)^7 + 91(sT)^6 + 286(sT)^5 + 495(sT)^4 + 462(sT)^3 + 210(sT)^2 + 36sT + 1}$$

Kontrolle mit 9. Ordnungen : Eingabe in TI -> ca. $\frac{1}{2}$ Zigarette warten => Ergebnisse OK

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