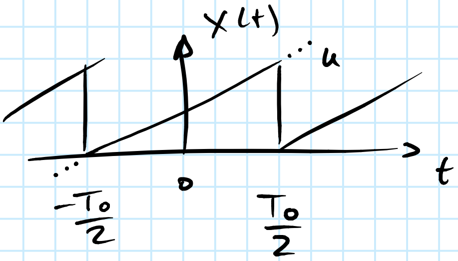
Beispiel: Fourierreihen, Sägezahn



$x(t)$ innerhalb Periode $= t \cdot \frac{u}{T_0} + \frac{u}{2}$

$$c_n = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-j\omega_0 n t}\, dt = \underbrace{\frac{1}{T_0}\int_{-\frac{T_0}{2}}^{+\frac{T_0}{2}} t\cdot\frac{u}{T_0}\, e^{-j\omega_0 n t}\, dt}_{I} + \underbrace{\frac{1}{T_0}\int_{-\frac{T_0}{2}}^{+\frac{T_0}{2}} \frac{u}{2}\, e^{-j\omega_0 n t}\, dt}_{II} =$$

$$I = \frac{u}{2T_0^2}\left(-\frac{t}{i\omega_0 n} + \frac{1}{\omega_0^2 n^2}\right) e^{-i\omega_0 n t}\Big|_{T_0} = \quad / \ (-i)^2 = +i^2 = -1$$

$$= \frac{u}{2T_0^2}\left(-\frac{\frac{T_0}{2}}{\frac{i2\pi n}{T_0}} + \frac{T_0^2}{4\pi^2 n^2}\right) e^{-i\pi n} - \frac{u}{2\cancel{T_0^2}}\left(+\frac{\cancel{\frac{T_0}{2}}}{\frac{i2\pi n}{\cancel{T_0}}} + \frac{\cancel{T_0^2}}{4\pi^2 n^2}\right) e^{i\pi n} =$$

$$= \frac{u}{2}\left(-\frac{1}{i\pi n} + \frac{1}{4\pi^2 n^2}\right) e^{-i\pi n} - \frac{u}{2}\left(\frac{1}{i\pi n} + \frac{1}{4\pi^2 n^2}\right) e^{i\pi n} = -\frac{u}{2\pi^2 n^2}\sin(\pi n) - \frac{u}{i\pi n}\cos(\pi n)$$

$$II = \frac{u}{2T_0}\cdot\frac{e^{-i\omega_0 n t}}{-i\omega_0 n} = \frac{u}{2T_0}\left(-\frac{e^{-i\pi t}}{i\omega_0 n} + \frac{e^{+i\pi t}}{i\omega_0 n}\right) =$$

$$= \frac{u\cdot 2i}{2T_0\, i\omega_0 n}\sin(\pi n) = \frac{u}{2\pi n}\sin(\pi n)$$

$$c[n] = u\left(\left(\frac{1}{2\pi n} - \frac{1}{2\pi^2 n^2}\right)\cdot\sin(\pi n) - \underbrace{\frac{1}{i\pi n}\cos(\pi k)}_{?}\right.$$

$$2i\cdot\sin(x) = e^{ix} - e^{-ix}$$
$$2\cdot\cos(x) = e^{ix} + e^{-ix}$$