Voltage Comparator and "Q" Multiplier make a stable Sine Wave Oscillator

Ramón Vargas Patrón
rvargas@inictel.gob.pe
INICTEL

It is a known fact that sinusoidal oscillators must rely on the action of some amplitude-limiting mechanism in order to achieve good frequency and amplitude stabilities. Likewise, accurate loop-gain control and high-Q tuned circuits are required for the attainment of low-distortion output levels.

This article discusses an OP-AMP based sine wave L-C oscillator that can be built using non-critical components and is capable of delivering a clean and stable output waveform. Fig.1 shows the schematic diagram of the oscillator.

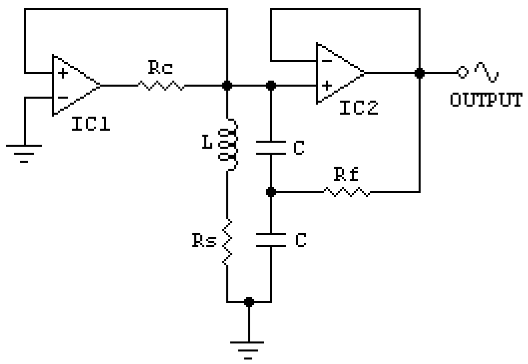


Fig. 1 Sine wave IC Oscillator

The operation principle is as follows: a high-gain non-inverting amplifier configured as a voltage comparator, IC_1, imposes some negative resistance across the parallel lossy resonant circuit. As soon as an oscillation builds up, a constant-amplitude square-wave signal source starts driving the tuned network, giving rise to a sine-like waveform across its terminals.

A second stage, IC_2, acts as a "Q" multiplier. It supplies additional negative resistance to the tank, raising its "Q" up to the point where harmonics of the square-wave signal's fundamental frequency are strongly attenuated, yielding a clean sine wave across the tuned circuit. The feedback resistor R_f is made variable for easy adjustment of the tank's "Q". The waveform's amplitude and frequency remain very stable after the setting has been effected.

Circuit analysis

As usual, we would like to arrive to formulae for the frequency and amplitude of oscillation, and for the conditions to be met for oscillations to occur. Before beginning our analysis we shall recall a simple network transformation that concerns to a lossy coil and whose proof is given in Appendix A.

It states that at any given radian frequency $\omega = 2\pi f$, a lossy inductor L can be made equivalent to a lossless coil L_P in parallel with a loss R_P:

$$L_P = L\left(1 + \frac{1}{{Q_S}^2}\right) \qquad \ldots(1.1)$$

$$R_P = R_S\left(1 + {Q_S}^2\right) \qquad \ldots(1.2)$$

where $Q_S = \dfrac{\omega L}{R_S}$ is the lossy inductor's "Q" factor at frequency ω and R_S is the series loss.

Applying eqs. (1.1) and (1.2) to the circuit of Fig.1 we arrive to the equivalent oscillator of Fig.2, which will be used for our analysis from here on.

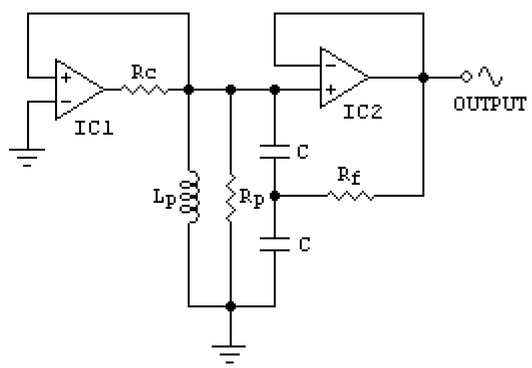


Fig. 2 Equivalent oscillator for analysis

It is interesting to notice that according to the above formulae, the resonant frequency ω_r of an isolated tuned circuit comprising the lossy coil L and the series-connected C capacitors is:

$$\omega_r = 2\pi f_r = \frac{1}{\sqrt{L\left(1 + \frac{1}{{Q_S}^2}\right)C_{eq}}} \qquad \ldots(2.1)$$

$$= \sqrt{\frac{1}{LC_{eq}} - \left(\frac{R_S}{L}\right)^2} \qquad \ldots(2.2)$$

where $C_{eq} = C/2$ and $Q_S = \dfrac{\omega_r L}{R_S}$.

Generation of negative resistances

The comparator stage comprising IC_1 (Fig.3-a) has the small-signal equivalent circuit shown in Fig.3-b. Here, A_d is the complex open-loop gain of the frequency-compensated OP-AMP, and is given by:

$$A_d = \frac{A_0}{1 + j\dfrac{f}{f_0}} \qquad \text{…(3)}$$

where A_o is the differential voltage gain at DC frequencies and f_o is the low-frequency pole of the gain plot (Fig.3-c).

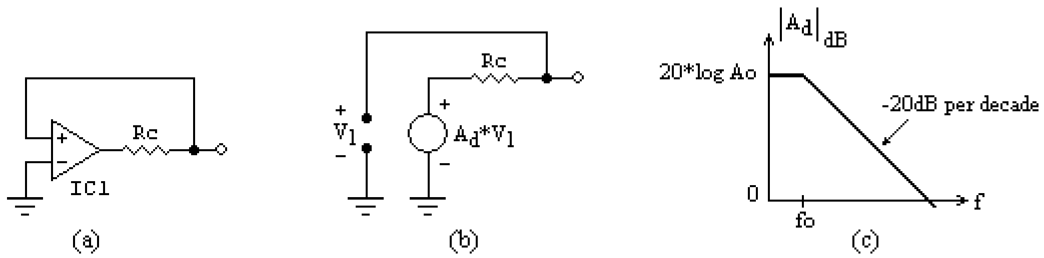


Fig.3 Comparator stage, small-signal equivalent circuit and OP-AMP's Bode gain plot

Let´s apply a test signal $v_t = V_T\,cos\omega t$ to the output port of the comparator. The current supplied by the test generator will be, according to Fig.3-b:

$$I_T = \frac{V_T - A_d V_T}{R_C}$$

$$= \frac{V_T\left(1 - A_d\right)}{R_C}$$

The quotient V_T / I_T is the small-signal output impedance Z_0 of the comparator:

$$Z_0 = \frac{V_T}{I_T} = \frac{R_C}{1 - A_d} \qquad \text{…(4.1)}$$

Z_0 acts as a source impedance and for resonance calculations it will appear across the terminals of the tuned circuit of Fig.2. The reader can verify that Z_0 is the parallel combination of R_C and $-\dfrac{R_C}{A_d}$, that is to say,

$$Z_0 = R_C \,//\left(-\frac{R_C}{A_d}\right) \qquad \text{…(4.2)}$$

or

$$Z_0 = R_C // \left[-\frac{R_C}{A_0}\left(1 + j\frac{f}{f_0}\right)\right] \qquad \ldots(4.3)$$

The real part of $-\frac{R_C}{A_d}$ is a negative resistance:

$$r_N = -\frac{R_C}{A_0} \qquad \ldots(5.1)$$

The imaginary component suggests a capacitive reactance, due to the negative sign:

$$X_N = -\frac{R_C f}{A_0 f_0} = -\frac{R_C \omega}{A_0 \omega_0} \qquad \ldots(5.2)$$

More over, it may be associated to a capacitance:

$$C_N = -\frac{1}{X_N \omega} = \frac{A_0 \omega_0}{R_C \omega^2} \qquad \ldots(5.3)$$

Clearly, r_N and C_N are series-connected. They can be transformed to their parallel equivalent using formulae for Case II from Appendix A (Fig.4):

$$R_{Pc} = -\frac{R_C}{A_0}\left[1 + \left(\frac{\omega}{\omega_0}\right)^2\right] \qquad \ldots(6.1)$$

$$C_{Pc} = \frac{A_0 \omega_0}{R_C \omega^2 \left[1 + \left(\frac{\omega_0}{\omega}\right)^2\right]}$$

$$= \frac{A_0}{\omega_0 R_C \left[1 + \left(\frac{\omega}{\omega_0}\right)^2\right]} \qquad \ldots(6.2)$$

where ω is any radian frequency of our interest.

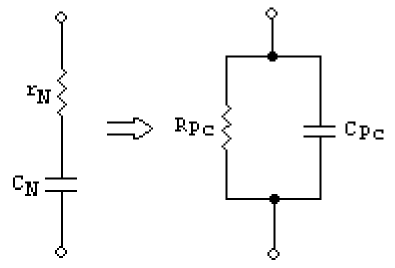


Fig.4 Series to parallel equivalent

Now, let´s turn our attention towards the "Q"-multiplier stage comprising IC_2. Appendix B shows that this stage supplies a negative resistance across the tank circuit given by:

$$R_{Pm} = -\frac{1}{\omega^2 C^2 R_f}\left(1 + 4\omega^2 C^2 R_f^{\ 2}\right) \qquad \ldots(7.1)$$

and modifies the tuning capacitance according to:

$$C_{Pm} = \frac{C_{eq}}{\left(1 + \frac{1}{4\omega^2 C^2 R_f^{\ 2}}\right)} \qquad \ldots(7.2)$$

where

$$C_{eq} = \frac{C}{2} \qquad \ldots(7.3)$$

Barkhausen's criterion for oscillation from the negative resistance point of view

So far, two important conclusions are:

1. Net tuning capacitance $\qquad C_T = C_{Pc} + C_{Pm} \qquad \ldots(8.1)$

2. Net parallel loss $\qquad R_T = R_C \,//\, R_P \,//\, R_{Pc} \,//\, R_{Pm}$

or

$$\frac{1}{R_T} = \frac{1}{R_C} + \frac{1}{R_P} + \frac{1}{R_{Pc}} + \frac{1}{R_{Pm}} \qquad \ldots(8.2)$$

For oscillations to build up, the net parallel loss should be infinity, or equivalently,

$$\frac{1}{R_T} = 0$$

Then:

$$\frac{1}{R_C}+\frac{1}{R_P}+\frac{1}{R_{Pc}}+\frac{1}{R_{Pm}}=0 \qquad \ldots(9)$$

Being convenient for our discussion, we shall make

$$\frac{1}{R_L}=\frac{1}{R_P}+\frac{1}{R_{Pm}} \qquad \ldots(10.1)$$

Then, eq.(9) reads:

$$\frac{1}{R_C}+\frac{1}{R_L}+\frac{1}{R_{Pc}}=0 \qquad \ldots(10.2)$$

or R_{Pc} = - (R_C // R_L).

From eqs. (6.1) and (10.2) we arrive to a very useful relationship:

$$\frac{A_0}{\left[1+\left(\frac{\omega_{osc}}{\omega_0}\right)^2\right]}=1+\frac{R_C}{R_L} \qquad \ldots(11)$$

where ω_{osc} is the frequency of oscillation, obtained from

$$\omega_{osc}{}^2=\frac{1}{L_P C_T} \qquad \ldots(12)$$

after recalling eqs.(1.1) and (8.1). Using eq.(6.2) and the equivalence shown by eq.(11) we arrive to an alternate form for C_{Pc}:

$$C_{Pc}=\frac{1+\frac{R_C}{R_L}}{\omega_0 R_C} \qquad \ldots(13)$$

Eq.(12) may now be written as:

$$\omega_{osc}{}^2=\frac{1}{L_P\left[\frac{1+\frac{R_C}{R_L}}{\omega_0 R_C}+C_{Pm}\right]}$$

or

$$\omega_{osc}{}^{2} = \frac{1}{L_P C_{Pm}\left[1+\dfrac{1+\dfrac{R_C}{R_L}}{\omega_0 R_C C_{Pm}}\right]} \qquad \ldots(14)$$

Important observations

1. If $Q_S = \dfrac{\omega_{osc}L}{R_S} > 10$, then $L_P = L$ within 1%. This condition can be met adjusting R_f for a clean waveform across the tuned circuit. Negative resistance effectively cancels circuit losses.
2. R_L can be made to be a negative value if R_f in the "Q"- multiplier stage is adjusted so that $\dfrac{1}{R_P} + \dfrac{1}{R_{Pm}} < 0$, which in turn will be an indication of complete coil-loss cancellation.
3. If $4\omega_{osc}{}^{2}C^{2}R_f{}^{2} >> 1$, then $R_{Pm} \approx -4R_f$ and $C_{Pm} \approx C_{eq} = \dfrac{C}{2}$.
4. Any change in R_L will yield a corresponding variation in ω_{osc}.

Calculation of the amplitude of oscillation

Eq. (11) tells us that the comparator's gain at the fundamental frequency will have to reduce itself, due to the amplitude-limiting mechanism, to the value:

$$\frac{A_0}{\sqrt{1+\left(\dfrac{\omega_{osc}}{\omega_0}\right)^2}} = \sqrt{1+\left(\dfrac{\omega_{osc}}{\omega_0}\right)^2}\left(1+\frac{R_C}{R_L}\right) \qquad \ldots(15)$$

The amplitude of the oscillation across the tank will be then:

$$X_1 = \frac{4V_1}{\pi}\cdot\frac{1}{\sqrt{1+\left(\dfrac{\omega_{osc}}{\omega_0}\right)^2}}\cdot\frac{1}{\left(1+\dfrac{R_C}{R_L}\right)} \qquad \ldots(16)$$

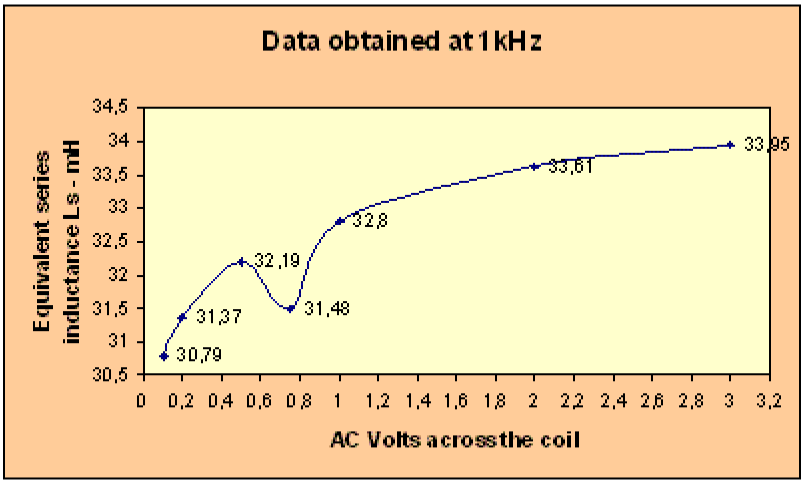
where $\dfrac{4V_1}{\pi}$ is the amplitude of the fundamental-frequency component of the square-wave output of the comparator swinging between $-V_1$ and $+V_1$ Volts.

Experimental results

An oscillator using the proposed topology was built around two independent TL072 ICs having low-frequency poles at f_0 = 20Hz and saturating output voltages of +/-11 Volts when energized from +/-12V DC split power supplies.

A magnetic telephone earpiece exhibiting 31mH and 60 ohms DC resistance on a B&K Precision 875A LCR-Meter was used in conjunction with two series-connected 0.33uF Mylar capacitors for the tank circuit. R_C was selected to be 10k ohms and R_f was a 1k-ohm variable resistor. The latter was adjusted for a 5-Volt peak oscillation across the tank. The frequency of the oscillation was measured as 2.181kHz, yielding a low-distortion and stable sine wave. Frequency drift was less than 2Hz in a period of an hour.

Careful measurements were conducted in order to check for the accuracy of the derived formulae. The equivalent L_S-R_S series circuit of the magnetic telephone-hearing element was obtained at different signal levels using a General Radio Type 1608-A Impedance Bridge. Readings were first taken at 1kHz using an external signal source for the bridge. A second set of measurements was obtained at 2.181kHz. In both cases, readings were taken at 0.1, 0.2, 0.5, 0.75, 1, 2 and 3Volts peak across the coil. Measured data is shown in Fig.5 below.



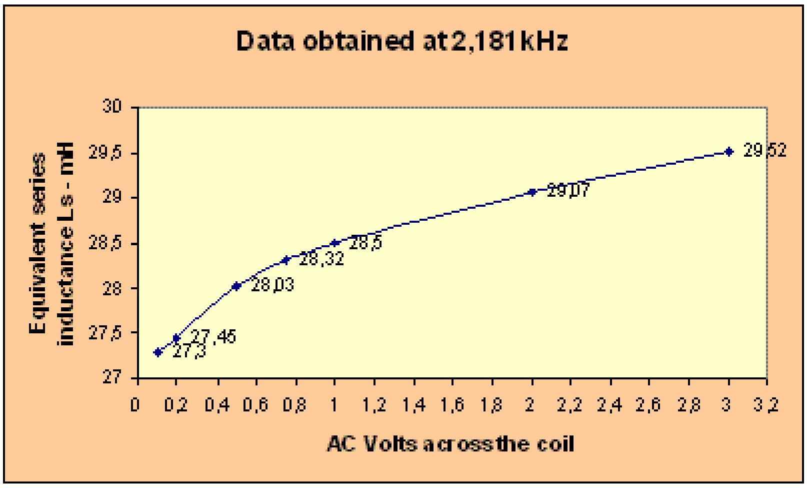


Fig.5 Influence of signal strength and frequency on the equivalent series inductance L_S of the magnetic hearing element

The restricted output of the available external signal generator impeded readings above 3 Volts. So, an additional set of measurements was taken with the circuit oscillating at peak tank-voltage amplitudes of 3, 4, 5, 6, and 6.9 Volts (last value before clipping started).

Eq.(7.1) permitted calculation of the negative resistance contributed by the "Q" multiplier stage for each value of signal amplitude. The contributed capacitance C_{Pm} was obtained from eq.(7.2). Table I below was constructed with this data and the corresponding measured values for the frequency of oscillation f_{osc} and feedback resistor R_f. Values for $1+\frac{R_C}{R_L}$ were calculated substituting the figures of the first two columns into eq.(16). Equation (14) was then solved for L_P and the values of R_P computed from eq.(10.1).

TABLE I

Tank voltage Volts	f_{osc} Hz	R_f ohms	R_{Pm} ohms	C_{Pm} nF	L_P mH	$1 + R_C/R_L$	R_P ohms
3.0	2133	396.4	-1714.57	152.59	29.70	0.0438	2050.80
4.0	2164	334.7	-1487.20	148.53	31.04	0.0323	1737.19
5.0	2181	308.6	-1392.85	146.23	31.95	0.0257	1611.55
6.0	2191	289.0	-1323.66	144.10	32.76	0.0213	1520.65
6.9	2196	278.2	-1286.17	142.76	33.36	0.0185	1471.99

It should be noticed that the calculated values for L_P shown on Table I will closely approximate the series inductance values L_S of the lossy coil L, due to the loss cancellation effected by the negative resistance contributed by the "Q"-multiplier stage. Fig.6 below shows the circuit that supplied values for Table I.

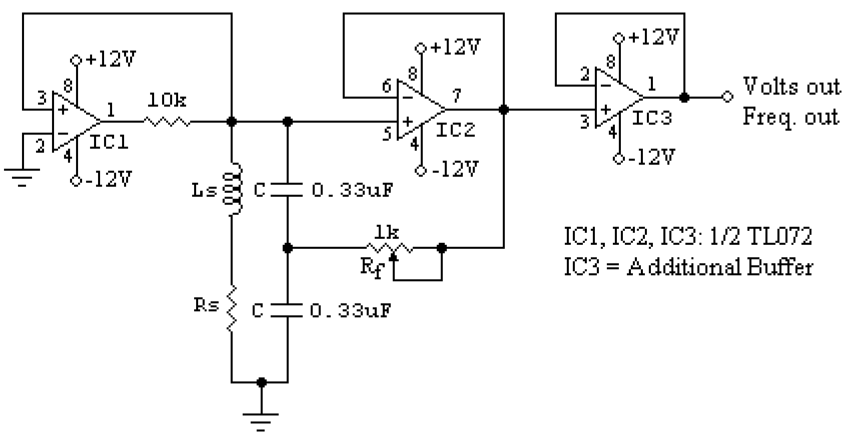


Fig.6 Oscillator circuit that supplied values for Table I

Conclusions

A sine wave L-C oscillator featuring excellent amplitude and frequency stabilities has been devised using a voltage comparator and a Colpitts-type "Q" multiplier. High performance is achieved thanks to the amplitude-stabilizing action of a voltage comparator stage that drives a lossy tank circuit with a constant-amplitude square wave signal. The "Q" multiplication provided by the Colpitts-type stage renders an output waveform with low harmonic content.

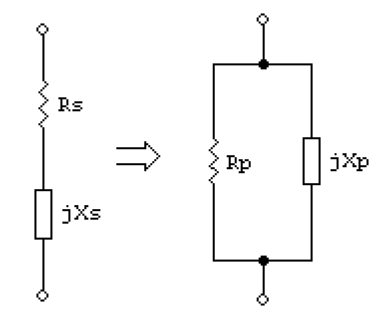
Formulae have been derived for the frequency and the amplitude of the oscillation across the tank circuit. Higher operation frequencies may be obtained using high-speed OP-AMPs, while maintaining the excellent stabilities observed at audio frequencies.

One straightforward application for the circuit could be the indirect determination of the equivalent series inductance L_S and parallel loss R_P of iron-cored inductors at different signal levels and specific operation frequencies. A second and obvious application is that of a stable frequency source.

Appendix A

Series to parallel transformation

We wish to put the impedance Z = R_S + jX_S into its parallel equivalent $Y = \frac{1}{R_P} + \frac{1}{jX_P}$.



Series to parallel transformation

The following is then true:

$$Y = \frac{1}{R_S + jX_S}$$

$$= \frac{R_S - jX_S}{R_S^{\,2} + X_S^{\,2}}$$

$$= \frac{R_S}{R_S^{\,2} + X_S^{\,2}} - j\frac{X_S}{R_S^{\,2} + X_S^{\,2}}$$

Let us define $Q_S = \left|\frac{X_S}{R_S}\right|$, i.e., the absolute value of $\frac{X_S}{R_S}$. Then:

$$Y = \frac{1}{R_S\left(1+Q_S^{\,2}\right)} + \frac{1}{jX_S\left(1+\frac{1}{Q_S^{\,2}}\right)}$$

$$= \frac{1}{R_P} + \frac{1}{jX_P}$$

Thus, we get the transformation pair:

$$R_P = R_S\left(1+Q_S^{\,2}\right)$$

$$X_P = X_S\left(1+\frac{1}{Q_S^{\,2}}\right)$$

X_P has the same sign as X_S.

Case I: Series R_S and L_S

$X_S = \omega L_S$ $R_S = R_S$	transforms to	$X_P = \omega L_S\left(1+\frac{1}{Q_S^{\,2}}\right)$ $R_P = R_S\left(1+Q_S^{\,2}\right)$

Then, series R_S and L_S transforms to parallel $L_P = L_S\left(1+\frac{1}{Q_S^{\,2}}\right)$ and $R_P = R_S\left(1+Q_S^{\,2}\right)$, with $Q_S = \frac{\omega L_S}{R_S}$.

Case II: Series R_S and C_S

$$X_S = -\frac{1}{\omega C_S} \qquad \text{transforms to} \qquad X_P = -\frac{1}{\omega C_S}\left(1+\frac{1}{{Q_S}^2}\right)$$

$$R_S = R_S \qquad\qquad\qquad\qquad R_P = R_S\left(1+{Q_S}^2\right)$$

Then, series R_S and C_S transforms to parallel $C_P = \dfrac{C_S}{1+\dfrac{1}{{Q_S}^2}}$ and $R_P = R_S\left(1+{Q_S}^2\right)$,

with $Q_S = \dfrac{1}{\omega C_S R_S}$.

<u>Appendix B</u>

Negative resistance generation in the "Q" multiplier

Fig.1 recalls the capacitive tapping in the "Q"-multiplier section of the oscillator, the top end of C_2 and right end of R_f being at the same potential "E" with respect to ground.

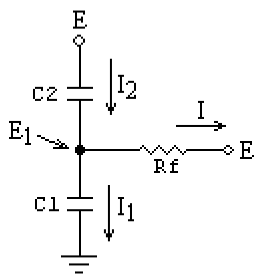


Fig.1 Capacitive tapping in the "Q"-multiplier

If we call E_1 the potential at the junction of C_1 and C_2, we may write:

$$I_2 = \left(E - E_1\right) j\omega C_2 \qquad \text{…(1.a)}$$

$$I_1 = E_1 j\omega C_1 \qquad \text{…(1.b)}$$

$$I = \frac{E_1 - E}{R_f} \qquad \text{…(1.c)}$$

$$I_1 = I_2 - I \qquad \text{…(2)}$$

Substituting eqs.(1.a), (1.b) and (1.c) into eq.(2):

$$(E - E_1) j\omega C_2 - \frac{(E_1 - E)}{R_f} = E_1 j\omega C_1$$

$$E\left(j\omega C_2 + \frac{1}{R_f} \right) = E_1 \left[j\omega (C_1 + C_2) + \frac{1}{R_f} \right]$$

Then:

$$E_1 = E \frac{j\omega C_2 + \dfrac{1}{R_f}}{j\omega (C_1 + C_2) + \dfrac{1}{R_f}} \qquad \ldots(3)$$

Eq.(1.a) in terms of eq.(3) gives:

$$I_2 = \left\{ E - E\left[\frac{j\omega C_2 R_f + 1}{j\omega (C_1 + C_2) R_f + 1} \right] \right\} j\omega C_2$$

$$= E\left[\frac{j\omega C_1 R_f}{j\omega (C_1 + C_2) R_f + 1} \right] j\omega C_2$$

$$= E\left[\frac{-\omega^2 C_1 C_2 R_f}{j\omega (C_1 + C_2) R_f + 1} \right]$$

Then:

$$\frac{E}{I_2} = -\frac{j\omega (C_1 + C_2) R_f + 1}{\omega^2 C_1 C_2 R_f}$$

$$= -\frac{j(C_1 + C_2)}{\omega C_1 C_2} - \frac{1}{\omega^2 C_1 C_2 R_f}$$

$$= \frac{1}{j\omega \left(\dfrac{C_1 C_2}{C_1 + C_2} \right)} - \frac{1}{\omega^2 C_1 C_2 R_f} \qquad \ldots(4)$$

The above expression suggests the existence of a negative resistance $R_S = -\dfrac{1}{\omega^2 C_1 C_2 R_f}$

in series with a capacitance $C_S = \frac{C_1 C_2}{C_1 + C_2}$. The series network can be transformed into a parallel equivalent (Fig.2). If we make:

$$R_S = -\frac{1}{\omega^2 C_1 C_2 R_f}$$

and

$$X_S = -\frac{1}{\omega\left(\frac{C_1 C_2}{C_1 + C_2}\right)}$$

then $Q_S = \left|\frac{X_S}{R_S}\right| = \omega(C_1 + C_2)R_f$, and we readily obtain:

$$R_P = -\frac{1}{\omega^2 C_1 C_2 R_f}\left[1 + \omega^2 (C_1 + C_2)^2 R_f{}^2\right] \qquad \ldots(5.1)$$

$$X_P = -\frac{1}{\omega\left(\frac{C_1 C_2}{C_1 + C_2}\right)}\left[1 + \frac{1}{\omega^2 (C_1 + C_2)^2 R_f{}^2}\right] \qquad \ldots(5.2)$$

and

$$C_P = \frac{\frac{C_1 C_2}{C_1 + C_2}}{\left[1 + \frac{1}{\omega^2 (C_1 + C_2)^2 R_f{}^2}\right]} \qquad \ldots(5.3)$$

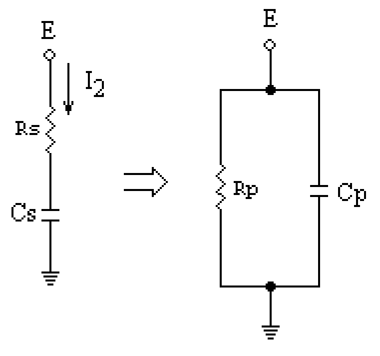


Fig.2 Series to parallel transformation

If $\omega^2 (C_1 + C_2)^2 R_f^{\ 2} >> 1$, eqs.(5.1), (5.2) and (5.3) reduce to:

$$R_P = -\frac{(C_1 + C_2)^2}{C_1 C_2} R_f$$

$$X_P = -\frac{1}{\omega \left(\frac{C_1 C_2}{C_1 + C_2} \right)}$$

$$C_P = \frac{C_1 C_2}{C_1 + C_2}$$

For the special case where $C_1 = C_2 = C$:

$$R_P = -4 R_f$$

$$C_P = \frac{C}{2}$$

Ramon Vargas Patron
rvargas@inictel.gob.pe
Lima-Peru, South America
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