Reciprocal maximum-length sequence pairs for acoustical dual source measurements[a]

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In this paper we propose and demonstrate a method to obtain simultaneous dual source–receiver impulse responses in acoustical systems using binary maximum-length sequences (MLS). A binary MLS and its reversed-order sequence form a reciprocal MLS pair. Their correlation property includes a two-valued "pulse-like" autocorrelation function and a relatively smaller-valued cross-correlation function. This unique property, along with other number-theory properties, makes the reciprocal MLS pair suitable for simultaneous dual source cross-correlation measurements. In the measurement of a dual source system, each of the reciprocal MLS pairs simultaneously excite one of two separate sources, one or several receiver signals cross-correlate in turn with each of the MLS pairs, resulting in impulse responses associated with two separate sources. The proposed method is particularly valuable for system identification tasks with multiple sound/vibration sources and receivers that have to be accomplished in a limited time period. A fast algorithm called a fast MLS transform is exploited for the cross-correlation. In this paper we propose a fast MLS transform pair for the reciprocal MLS pairs. Its efficiency lies in the requirement of one single permutation matrix for a pair of two fast MLS transforms. Its feasibility and usefulness in the acoustical measurements are demonstrated using experimental results. [DOI: 10.1121/1.1561498]

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I. INTRODUCTION

A wide acceptance of binary maximum-length sequences (MLS or M sequences) measurement technology in the acoustics community is due to MLS' excellent number-theory properties. One of the key characteristics of the MLS is their two-valued periodic autocorrelation function. It is this autocorrelation property that is exploited in most of the applications of MLS. The advanced MLS measurement technique is based on a fast algorithm termed a *Fast M-sequence Transform* (FMT) by Cohn and Lempel[1] in which the impulse response-related system identification tasks can be accomplished efficiently. In addition, MLS, as excitation signals, possess a high signal power and low peak factors. Along with the inherent cross-correlation mechanism for system identification, a high noise immunity in measurement results can be obtained. Recent acoustical applications of the MLS measurement technique can be found, among others, in architectural acoustics,[2–7] audiology,[8–10] ultrasonics,[11,12] psychoacoustics,[13,14] underwater acoustics,[15] and physical acoustics.[16,17]

In this paper we propose a technique using a reciprocal pair of MLS in simultaneous dual-source channel measurements. Impulse responses between two separate sources and one or several receivers of acoustical systems can be determined simultaneously. The simultaneous dual-source measurements exploit the cross-correlation properties of reciprocal MLS pairs that are considerably less widely known and understood than the autocorrelation functions. It is this property that makes simultaneous dual-source measurements feasible. This technique is of practical significance for a number of acoustical investigations in physical acoustics, ultrasonics, and architectural acoustics. Particularly, some measurement tasks of an acoustical system under test with multiple sound/vibration sources have to be accomplished in a limited time period. In Sec. II we briefly introduce some number-theory properties pertaining to the technique. In Sec. III we then describe a convenient algorithm for the FMT and in Sec. IV derive a permutation matrix for the reciprocal MLS pairs. In Sec. V we discuss some acoustical experiments designed for a demonstration of the usefulness of the properties and the convenience of the algorithm.

II. BASIC PROPERTIES OF BINARY MLS

An n-stage linear feedback shift-register device can generate a binary periodic sequence $\{a_i\}$ with $a_i \in \{0,1\}$. When its feedback taps are appropriately connected, the periodic sequences arrive at their maximum period length of $L = 2^n - 1$. In this case, the sequences are referred to as maximum-length sequences (MLS). A characteristic polynomial $f(x)$ expressing its feedback connection is then referred to as *primitive*. The positive integer n is said to be the degree of the MLS as well as its primitive polynomial (PP). In math-

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ematical treatment MLS are convenient in their binary form $a_i \in \{0,1\}$ while a bipolar form $m_i \in \{-1,+1\}$ is often used in practice to generate waveforms with $m_i = 1 - 2a_i$.

MLS enjoy a number of attractive properties that make them widely useful in broad scientific and engineering fields. In this section we briefly review some basic properties pertaining to the following discussion with respect to the application of reciprocal MLS pairs. A detailed description and definitions can be found in Refs. 18–23.

A. Decimation of MLS

If $\{a_i\}$ is an MLS of length $L = 2^n - 1$, a decimation $\{a_{qi}\}$ of sequence $\{a_i\}$ yields another MLS $\{b_i\}$ of the same degree with $b_i = a_{qi}$, if and only if (see Ref. 18) the greatest common divider of positive integers q and L equals one, $\gcd(q,L) = 1$. q is said to be a proper decimation factor. The index operation $q \cdot i$ of $\{a_{qi}\}$ and all others throughout this paper are evaluated modulo L.

A decimation factor of 2 results in the same MLS $\{b_i\}$ with $b_i = b_{2i}$ only for a unique initial state of the linear feedback shift register for every individual PP $f(x)$. This special class of MLS is designated as characteristic,[25] also as an idempotent,[19] self-similar[26] MLS. Without restriction on a specific initial state, a decimation factor of 2 or even 2^k will generally yield a phase-shifted MLS associated with the same PP,[18] with k being a positive integer. Recently, Xiang *et al.*[26] described an algorithm for determining the initial state according to Gold.[25] The self-similar, characteristic MLS substantiated in terms of this invariant decimation have found applications in ultrasonic measurement techniques.[26] They are also of practical significance for the technique proposed in this paper.

With some other proper decimation factors, decimation may yield distinctly different MLS generated by distinctly different PP of the same degree. This variant decimation property is worth mentioning together with reciprocal MLS pairs and their cross-correlation property in the following.

B. Primitive elements over GF(2^n) and trace orthogonal basis

A primitive polynomial (PP) $f(x) = \Sigma_{j=0}^{n} c_j x^j$ exactly expressing the feedback connections of the linear feedback shift register with its coefficients c_j over a finite field, called a Galois field GF(2), has a close connection with the elements of GF(2^n). Given a PP $f(x)$ of degree n, one can always find an element ν of GF(2^n) such that $f(\nu^{-1}) = 0$.[20] ν is a primitive element of GF(2^n) and so is ν^{-1}, namely $f(\nu) = 0$. The characteristic MLS has a close relation to the trace operator of an element α over GF(2^n).[20] The trace of an arbitrary element $\alpha \in \mathrm{GF}(2^n)$ is defined by

$$\mathrm{Tr}(\alpha) = \sum_{k=0}^{n-1} \alpha^{2k}. \tag{1}$$

And a basis $\Omega = \{\omega_0, \omega_1, \ldots, \omega_{n-1}\}$ of GF(2^n) over GF(2) is termed the trace orthogonal basis (TOB) if

$$\mathrm{Tr}(\omega_i) = 1, \quad \omega_i \in \Omega \tag{2}$$

and

$$\mathrm{Tr}(\omega_i \omega_j) = 0, \quad \omega_i, \omega_j \in \Omega, \quad i \neq j. \tag{3}$$

A TOB can be considered as a special coordinate system containing n vectors $\omega_0, \omega_1, \ldots, \omega_{n-1}$ that are orthogonal to each other in a sense of the trace operator. The practical significance of a TOB for GF(2^n) lies in the fact that an arbitrary element $\nu^i \in \mathrm{GF}(2^n) - \{0\}$ can be represented by an n-tuple binary number $\{e_{i0}, e_{i1}, \ldots, e_{i(n-1)}\}$ (see Ref. 7 for more details) as

$$\nu^i = \sum_{j=0}^{n-1} e_{ij}\omega_j; \quad 0 \leqslant i < L \tag{4}$$

and

$$\mathrm{Tr}(\nu^i \nu^j) = \sum_{k=0}^{n-1} e_{ik} e_{jk}, \tag{5}$$

with

$$e_{ij} = \mathrm{Tr}(\nu^i \omega_j) \in \mathrm{GF}(2). \tag{6}$$

Equations (4)–(6) will be useful for the derivation in Sec. IV.

C. Reciprocal MLS and polynomials

If $\{a_i\}$ is a MLS of length $L = 2^n - 1$ generated by $f(x)$, one can derive another $\{b_i\}$ of the same degree in terms of simply reverting the sequence with

$$b_i = a_{-i}. \tag{7}$$

$\{b_i\}$ is then generated by a primitive polynomial $r(x)$ derived by simply reverting the given PP $f(x)$ of degree n:

$$r(x) = x^n f(x^{-1}). \tag{8}$$

$r(x)$ is termed the reciprocal polynomial of $f(x)$. The MLS $\{b_i\}$ generated by $r(x)$ is termed reciprocal MLS of $\{a_i\}$. A pair of reciprocal PP $f(x)$, $r(x)$ always associate with a reciprocal pair of characteristic MLS $\{a_i\}$, $\{b_i\}$, respectively. In addition, if ν is a primitive element of $f(x)$ such that $f(\nu^{-1}) = 0$, then ν^{-1} is a primitive element of $r(x)$ such that $r(\nu) = 0$.[7]

A reciprocal MLS $\{b_i\}$ of $\{a_i\}$ can also be derived from the given $\{a_i\}$ in terms of decimation with a factor of $q = L - 1$. In effect, $b_i = a_{qi} = a_{(L-1)i} = a_{-i}$, which is exactly Eq. (7) since the index operation is evaluated modulo L. If $\{a_i\}$ is a characteristic MLS, $\{b_i\}$ achieved using Eq. (7) is also a characteristic MLS. The cross-correlation function of the reciprocal MLS pairs is of practical significance for the technique being discussed in the following.

D. Correlation property of MLS

MLS are periodic pseudorandom signals. The normalized periodic autocorrelation function (PACF) of a bipolar MLS within one period is a two-valued function[4] with

$$\phi(i) = \frac{L+1}{L}\,\delta(i) - \frac{1}{L}; \quad 0 \leqslant i < L. \tag{9}$$

When the period length $L = 2^n - 1$ is large enough, the PACF of MLS approximates a unit-sample sequence:

$$\phi(i) \approx \delta(i); \quad 0 \leqslant i < L. \tag{10}$$

From its PACF, it is readily deduced that its power spectral density function is of broadband nature and covers the entire frequency range (except for the zero frequency). It is this well-known property of MLS that has been exploited in most applications. Figure 1(a) illustrates the normalized PACF of a MLS of degree 12 for one period.

Cross-correlation properties of MLS are considerably less widely known, particularly in the acoustics community. Cross-correlation means correlation of one bipolar MLS with another of the same degree. There exists a number of MLS pairs for which their periodic cross-correlation functions (PCCF) possess considerably smaller values in comparison to the peak value of their two-valued PACF. Golomb[18] has observed that if $\{a_i\}$ and $\{b_i\}$ are generated by different PPs, then the PCCF ϕ_{ab} takes on at least three values. An appropriate decimation of MLS will yield pairs of MLS having a relatively small three-valued PCCF. The small valued PCCF depends only on the decimation factor rather than upon individual MLS. These three-valued PCCF of MLS pairs have gained considerable attention due to their (relatively) small magnitude in spread spectrum communication systems.[22] They are termed preferred pairs of MLS.[23] In addition to the preferred three-valued PCCF, there also exist four-valued PCCFs for specific classes of MLS.[22] They are even slightly better than the three-valued pairs with respect to the small magnitude. This paper will refer to both the three-valued and four-valued MLS pairs as preferred pairs. Appropriate decimation factors leading to the preferred MLS pairs are well documented and can be found in Refs. 22–23.

The most significant feature in this context is that the cross-correlation of the reciprocal MLS pairs also have small values being close to those of three-valued MLS pairs, yet not limited to three values.[22] The normalized cross-correlation bound $l(n)$ of the reciprocal MLS pairs is dependent upon the degree n:[22,24]

$$l(n) = \frac{2^{(n+2)/2} - 1}{2^n - 1}. \tag{11}$$

For the applications exploiting the small peak cross-correlation properties, one should not necessarily insist on having three- or four-valued PCCF. Figures 1(b) and 1(c) illustrate the normalized PCCF of a reciprocal MLS pair with a degree of 12 for one period. For convenience of comparison, the PACF of one of MLS is also illustrated in Fig. 1(a). Table I lists the peak cross-correlation values for degree 8–24 achieved by calculating the peak cross-correlation between these reciprocal MLS pairs compared with the cross-correlation bound values $l(n)$ predicted using Eq. (11).

III. FAST MLS TRANSFORM

Applying a periodic bipolar MLS $\{m_i\}$ to a linear time-invariant system under test and receiving one period of the system response $\mathbf{Y}$ to the MLS after the system arrives at its steady state, its impulse response $\mathbf{h}$ can predominantly be determined in terms of cross-correlation between the excitation MLS and the system response to the MLS[1,7] by

$$\mathbf{h} = \mathbf{M}\mathbf{Y}, \tag{12}$$

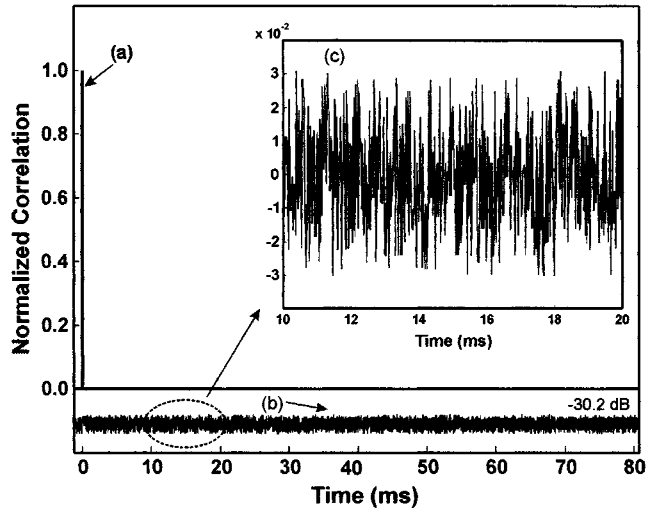

FIG. 1. Correlation functions of a MLS of degree 12 generated at sampling frequency of 50 kHz. (a) Autocorrelation function. (b) Cross-correlation function between the reciprocal MLS pair (shifted downward beneath the autocorrelation function for a convenient comparison while keeping the same amplitude scale). The peak value of the cross-correlation amounts to 0.03, 30.2 dB lower than the peak value of the autocorrelation. (c) A zoomed presentation of a segment from (b).

where $\mathbf{h}$, $\mathbf{Y}$ are vectors of L elements. $\mathbf{M}$ represents the MLS matrix of dimension $L \times L$, its rows contain sequentially right-cyclically shifted MLS $\{m_i\}$:

$$\mathbf{M} = [M_{ij}] = [m_{j-i}]; \quad i,j = 0,1,...,L-1. \tag{13}$$

Equation (12) is termed the *M sequence (MLS) transform*. Taking a bipolar $\{m_i\}$ of degree 3 as an example $\{m_i\} = \{-1,-1,-1,+1,-1,+1,+1\}$ and using its binary version $\{a_i\}$ for the MLS matrix, the binary MLS matrix $\mathbf{A}$ (binary version of $\mathbf{M}$) becomes

TABLE I. Cross-correlation bound values $l(n)$ [Eq. (11)] and normalized peak cross-correlation values of reciprocal MLS pairs achieved experimentally by performing the fast MLS transform of reciprocal MLS.

Degree	Period length	Cross-correlation bound [Eq. (11)]	Peak cross-correlation
8	255	0.1216	0.1216
9	511	0.0881	0.090
10	1023	0.0616	0.061
11	2047	0.0437	0.043
12	4095	0.0310	0.0308
13	8191	2.198E-2	0.0222
14	14 383	1.557E-2	0.0155
15	32 767	1.012E-2	1.013E-3
16	65 535	7.797E-3	7.78E-3
17	131 071	5.517E-3	5.51E-3
18	262 143	3.903E-3	3.90E-3
19	524 287	2.760E-3	2.76E-3
20	1 048 575	1.952E-3	1.95E-3
21	2 097 151	1.381E-3	1.38E-3
22	4 194 303	9.763E-4	9.76E-4
23	8 388 607	6.904E-4	6.90E-4
24	16 777 215	4.882E-4	4.88E-4

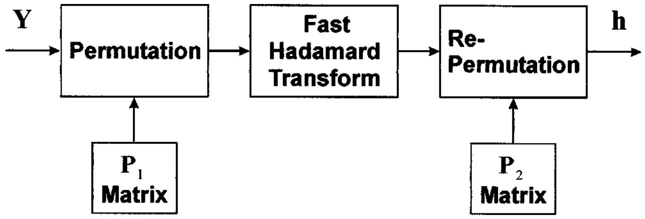

FIG. 2. Flow diagram of the Fast MLS Transform algorithm. A system response **Y** to the MLS undergoes permutation, Fast Hadamard Transform and repermutation, yielding the impulse response **h** directly in the time domain. In general, the algorithm requires two permutation matrices from the given MLS.

$$\mathbf{A}=[A_{ij}]=[a_{j-i}]=\begin{bmatrix}1&1&1&0&1&0&0\\0&1&1&1&0&1&0\\0&0&1&1&1&0&1\\1&0&0&1&1&1&0\\0&1&0&0&1&1&1\\1&0&1&0&0&1&1\\1&1&0&1&0&0&1\end{bmatrix}. \tag{14}$$

The MLS matrix in Eq. (13) is permutationally similar to Hadamard matrix:[23,27]

$$\mathbf{M}=\mathbf{P_2HP_1}, \tag{15}$$

where **H** is a Hadamard matrix of Sylvester-type. $\mathbf{P_1}$, $\mathbf{P_2}$ denote the permutation and the repermutation matrix, respectively. Equation (15) implies a fast algorithm, referred to as *Fast MLS Transform* (FMT) by Cohn and Lempel,[1] since the fast Hadamard transform for Hadamard matrix **H** is adopted in the calculation. Figure 2 illustrates a flow diagram for the FMT. It consists of three major steps.

(1) Permutation of the system response **Y** to the MLS being used as excitation of the system under test $(\mathbf{P_1Y})$.
(2) Fast Hadamard transform of the permuted vector $(\mathbf{H}(\mathbf{P_1Y}))$.
(3) Repermutation of the transformed vector $(\mathbf{P_2}[\mathbf{H}(\mathbf{P_1Y})])$.

An impulse response **h** results directly in the time domain right after the repermutation, except for a scale factor.[7]

Generally two permutation matrices are required by the fast MLS transform, as indicated in Eq. (15) and Fig. 2 (see, among others, Refs. 1, 4, 28). The two permutation matrices are derived from the MLS being used for the excitation signal. They are usually stored in form of indices.[7] Since the fast MLS transform performs inherently the cross-correlation between the MLS itself and the system response to the MLS, the two permutation matrices together can be considered as the original binary MLS in index form. In the following section we briefly describe an approach to construct the permutation matrices from the characteristic MLS. It yields one single permutation matrix that can be used at the same time for a pair of two reciprocal MLS.

IV. PERMUTATION MATRIX OF RECIPROCAL MLS PAIR

If $\{a_i\}$ is a characteristic MLS of degree n generated by a PP $f(x)$ and ν is a primitive element over $\mathrm{GF}(2^n)$, which satisfies $f(\nu^{-1})=0$, then the MLS matrix **A** can be factored using Eqs. (4)–(6) in terms of the trace operator and the TOB[7] as

$$\begin{aligned}\mathbf{A}=[A_{ij}]=[a_{j-i}]&=[\mathrm{Tr}(\nu^{-i}\nu^{j})]\\&=\left[\sum_{k=0}^{n-1} e_{(-i)k}e_{jk}\right]\\&=\mathbf{E}_2\mathbf{E}_1;\quad i,j=0,1,\ldots,L-1.\end{aligned} \tag{16}$$

A close investigation of Eq. (16) reveals that $\mathbf{E}_1$, $\mathbf{E}_2$ are of a similar structure in such a way that the rows of $\mathbf{E}_2$ can be found in the columns of $\mathbf{E}_1$ in the reversed order except for the first row and the first column. Taking the previous MLS of degree 3 in Eq. (14) as an example, $\mathbf{E}_1$ and $\mathbf{E}_2$ become

$$\mathbf{E}_1=\begin{bmatrix}1&1&0&1&0&0&1\\1&0&1&0&0&1&1\\1&0&0&1&1&1&0\end{bmatrix} \tag{17}$$

and

$$\mathbf{E}_2^T=\begin{bmatrix}1&1&0&0&1&0&1\\1&1&1&0&0&1&0\\1&0&1&1&1&0&0\end{bmatrix}, \tag{18}$$

where T stands for transpose of a matrix. Readers can easily verify that $\mathbf{E}_2\mathbf{E}_1$ results in the binary MLS matrix **A** in Eq. (14). Matrices $\mathbf{E}_1$, $\mathbf{E}_2$ lead straightforwardly to permutation matrices $\mathbf{P}_1$, $\mathbf{P}_2$, which can be expressed in index form by converting each binary column in Eqs. (17)–(18) into a decimal number with the top row containing the insignificant bit:[7]

$$\mathbf{P}_1=(7,1,2,5,4,6,3)_{\text{index}} \tag{19}$$

and

$$\mathbf{P}_2=(7,3,6,4,5,2,1)_{\text{index}}. \tag{20}$$

A detailed calculation of $\mathbf{P}_1$ can be found in Refs. 3, 7. In a practical implementation, once the permutation indices $\mathbf{P}_1$ is calculated from the given MLS, the repermutation (requiring $\mathbf{P}_2$) needs to take the indices from $\mathbf{P}_1$ in the reversed order, except for the first index while the permutation takes the indices in its sequential order. The FMT using this approach is illustrated in Fig. 3, where only one permutation matrix is required.

For a given PP $f(x)$, its reciprocal MLS $\{b_i\}$ of $\{a_i\}$ is generated by its reciprocal PP $r(x)=x^n f(x^{-1})$. If ν^{-1} satisfies $f(\nu^{-1})=0$, so does $(\nu^{-1})^{-1}$ satisfy $r[(\nu^{-1})^{-1}]=0$. In a similar fashion, the reciprocal MLS matrix **B** can be factored using Eqs. (4)–(6) as[7]

$$\begin{aligned}\mathbf{B}=[b_{j-i}]&=[\mathrm{Tr}((\nu^{-1})^{j-i})]\\&=[\mathrm{Tr}(\nu^{-j}\nu^{i})]=\left[\sum_{k=0}^{n-1} e_{ik}e_{(-j)k}\right]=\mathbf{G}_2\mathbf{G}_1.\end{aligned} \tag{21}$$

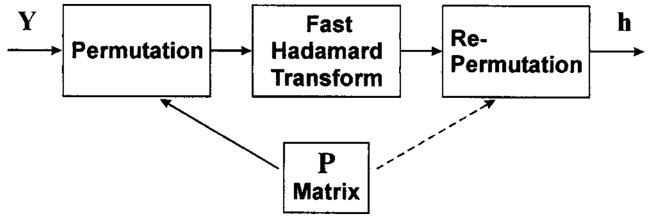

FIG. 3. Flow diagram of the Fast MLS Transform algorithm of a characteristic MLS. The algorithm requires only one permutation matrix. The dashed-line arrow from the permutation matrix to the repermutation implies that the repermutation takes the permutation indices in a reversed order while the permutation just takes its sequential order (solid arrow).

A close comparison of Eq. (21) with Eq. (16) reveals that $\mathbf{G}_1^T=\mathbf{E}_2$ and $\mathbf{G}_2=\mathbf{E}_1^T$, since the primitive element ν in Eq. (21) is the same as that in Eq. (16). This implies that the FMT for the reciprocal MLS $\{b_i\}$ uses the same single permutation matrix. Figure 4 shows the flow diagram of the FMT pair if the reciprocal MLS pair comes into practical applications that we elaborate on in the following section.

V. APPLICATIONS

In a variety of acoustical applications, a system under test can contain multiple sound/vibration sources and receivers, such as a listening situation in auditoria,[29] where, e.g., multiple loudspeakers of a sound system operate, or a number of musical instruments are played at the same time on the stage. Moreover, some impulse response-related system identification tasks with multiple sources have to be accomplished in a limited time period.[30,31] A simultaneous source measurement technique is especially needed.

A. Principle of simultaneous measurements

A system schematically illustrated in Fig. 5 is suitable for modeling the system identification tasks with multiple sources. In the figure, S_i denotes the ith source signal in the time domain while R_j denotes the jth receiver signal in the time domain. h_{ij} stands for an impulse response defined between the ith source and the jth receiver of a linear system.

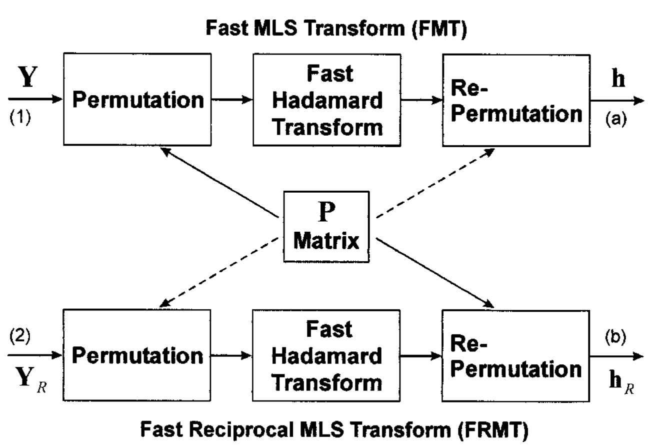

FIG. 4. Flow diagram of the Fast MLS Transform pair. The algorithms require only one permutation matrix for both the fast MLS transform (FMT) and the fast reciprocal MLS transform (FRMT). The dashed-line arrows from the permutation matrix to the permutation or repermutation imply that these permutations take the permutation indices in a reversed order while the other permutations just take their sequential order (solid arrows).

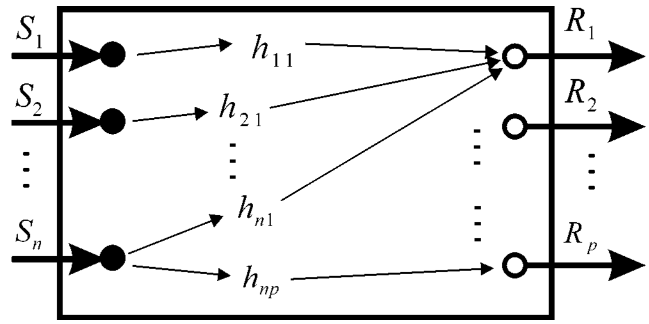

FIG. 5. System-theoretical model of a multisource and multireceiver system. All together, $n\times p$ impulse responses h_{ij} are defined between n sources and p receivers.

In the presence of all source signals as required by simultaneous source channel measurements, the jth receiver yields an output signal:

$$R_j(t)=\sum_{k=1}^{n} S_k(t)*h_{kj}(t);\quad j=1,...,p, \tag{22}$$

where "*" stands for the linear convolution, assuming that the system under test can be considered as a linear time-invariant system. A cross-correlation between the ith source signal and the jth receiver signal reads as

$$S_i(t)\otimes R_j(t)=\sum_{k=1}^{n} \phi_{ik}(t)*h_{kj}(t);$$
$$i=1,...,n;\quad j=1,...,p, \tag{23}$$

where $\otimes$ stands for linear cross-correlation and $\phi_{ik}(t)=S_i(t)\otimes S_k(t)$ for cross-correlation between the ith and kth source signal.

For experimentally determining the impulse responses, the following PCCF of excitation signals is desirable:

$$\phi_{ik}(t)=\begin{cases}\delta(t), & \text{for } i=k,\\ 0, & \text{for } i\neq k,\end{cases} \tag{24}$$

since inserting Eq. (24) into Eq. (23) yields

$$h_{ij}(t)=S_i(t)\otimes R_j(t);\quad i=1,...,n;\quad j=1,...,p. \tag{25}$$

Equation (25) indicates that the impulse response between the ith source and the jth receiver could conveniently be determined if the simultaneous excitation signals would possess the property expressed in Eq. (24), namely, their PACF would be a unit-sample sequence while their PCCF would equal zero. Lüke[23] pointed out that signals with exactly such a cross-correlation property can neither exist nor be constructed. Fortunately, some special classes of binary MLS are most suitable candidates for the discussed tasks due to the following two reasons.

First, the cross-correlation of reciprocal and preferred pairs of binary MLS discussed in Sec. II B approximates the desired condition in Eq. (24). As expressed both in Eqs. (10) and (11), illustrated in Fig. 1 and listed in Table I, the longer the sequences are to be used, the smaller the peak cross-correlation value becomes and the closer their cross-

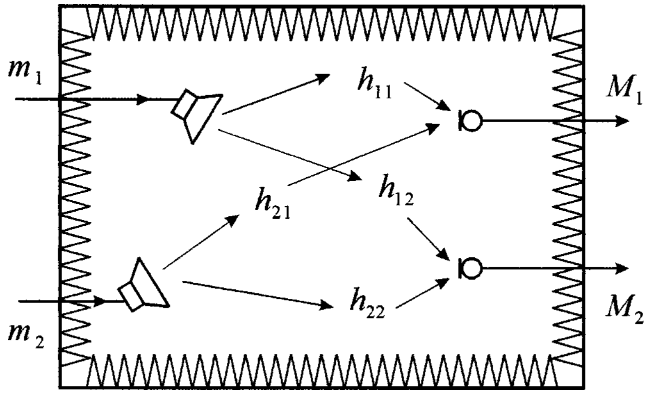

FIG. 6. Simultaneous acoustical measurement undertaken in an anechoic chamber with two sound sources and two microphones.

correlation approximate the condition in Eq. (24). Second, the FMT described in Sec. III can accomplish the operation expressed in Eq. (25) with high computational efficiency, particularly using the fast reciprocal MLS transform pair, as discussed in Sec. IV, two MLS (reciprocal pair) need only a single permutation matrix.

B. Experimental results

An exploratory experiment was carried out in an anechoic chamber, as schematically illustrated in Fig. 6, when a reciprocal MLS pair (m_1, m_2) excited two sound sources simultaneously. Each MLS of degree 13 at a sampling frequency of 30 kHz excited one sound source, respectively. Two microphones received one period of responses (M_1, M_2) to the MLS excitations after the system arrived at the steady state. Each microphone signal was then fed into input labeled by (1) of the FMT illustrated in Fig. 4 and then into the input labeled by (2) of the FRMT, respectively.

Since each microphone signal contained a linear combination of responses to the individual MLS from each sound source and the FMT pair performed the cross-correlation of each MLS to the microphone signal, the FMT [from input (1) to output (a) in Fig. 4] approximately filtered out the impulse response h_{11} and suppressed the component $m_2 * h_{21}$ with microphone signal M_1 feeding into the input labeled by (1) of the FMT. And the FRMT [from input (2) to output (b)] approximately filtered out the impulse response h_{21} and suppressed the component $m_1 * h_{11}$ with microphone signal M_1 feeding into the input labeled by (2) of the FRMT. One permutation matrix was determined using the given MLS of degree 13 for the pair of FMT and FRMT. In a similar fashion, when feeding microphone signal M_2, in turn, into the input labeled by (1) of the FMT and then into the input labeled by (2) of the FRMT, the FMT pair (in Fig. 4) approximately yielded h_{12} and h_{22}. Figure 7 shows the first 16 ms segments of four impulse responses. The peak-to-noise ratio of 30–31 dB was achieved for all four impulse responses, about 2–3 dB less than the peak cross-correlation value listed in Table I (for degree 13). A single-excitation impulse response with a peak-to-noise ratio of 53 dB, corresponding to h_{12}, but achieved from a single loudspeaker-microphone measurement using an M sequence of degree 13 is also plotted in Fig. 7 for comparison. This measurement

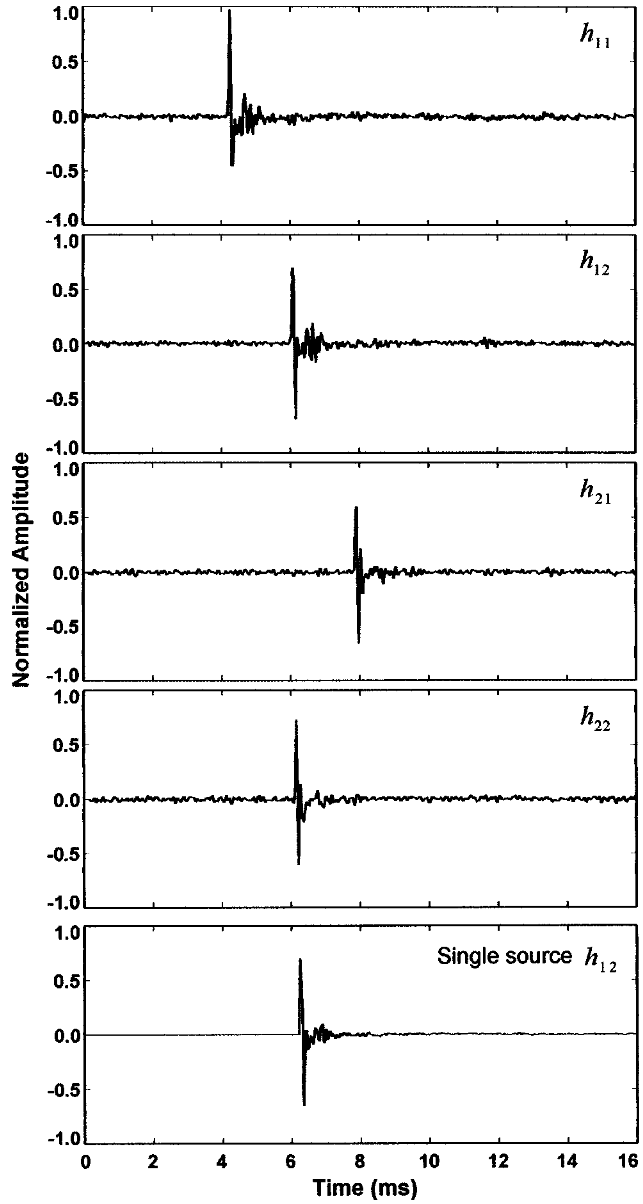

FIG. 7. Segments of impulse responses simultaneously measured in an anechoic chamber using a reciprocal MLS pair of degree 13 as excitation signals at the two (loudspeaker) source channels, respectively. Each of two microphone signals is, in turn, fed into two inputs of the FMT pair (illustrated in Fig. 4) yielding four impulse responses (h_{11}, h_{12}, h_{21}, and h_{22}). A single-source impulse response, corresponding to h_{12}, but achieved from a single loudspeaker-microphone measurement is also plotted for a comparison.

technique can be used, among others, for the study of outdoor sound propagation, as required by the experimental study in Refs. 30–31.

Another example is a simultaneous measurement of room impulse responses carried out in an auditorium (in the National Center for Physical Acoustics, The University of Mississippi) with a stereo sound system. The measurement setup is similar to the sketch in Fig. 6 but in the auditorium. A reciprocal MLS pair of degree 20, at a sampling frequency of 50 kHz, drove the pair of loudspeakers, respectively, and two microphones received one period of the responses to the simultaneous MLS excitations from both stereo loudspeakers. One permutation matrix was determined using the given

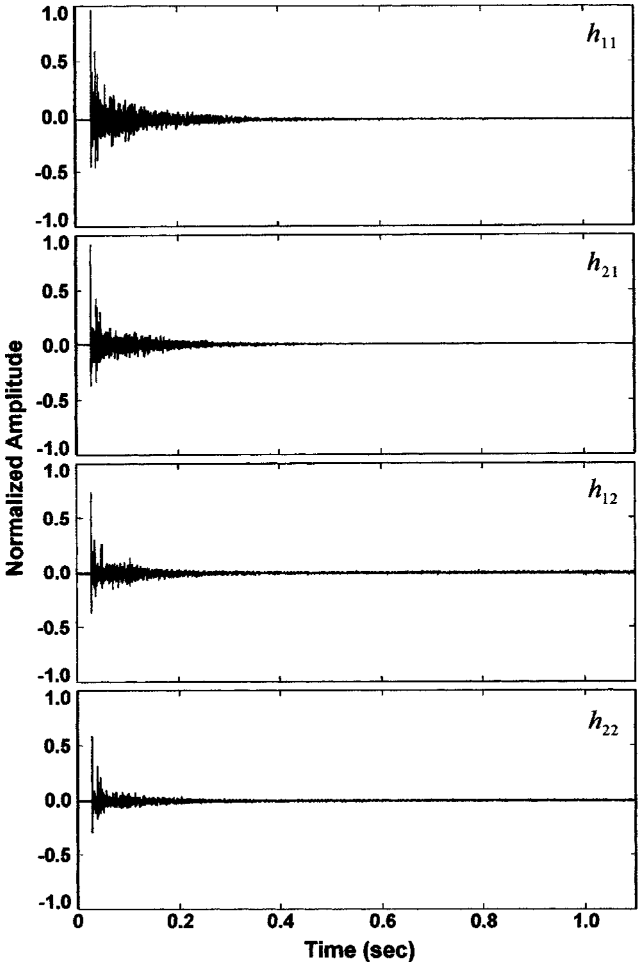

FIG. 8. Segments of room impulse responses simultaneously measured in an auditorium using a reciprocal MLS pair of degree 20 as excitation signals driving two stereo loudspeakers of a sound system, respectively. Two microphones are used for the sound receivers. Each of two microphone signals is, in turn, fed into two inputs of the FMT pair (illustrated in Fig. 4) yielding four room impulse responses (h_{11}, h_{12}, h_{21}, and h_{22}).

MLS of degree 20 for the FMT pair. Each of the two microphone signals was, in turn, fed into each input of the FMT pair, resolving two pairs of room impulse responses, as illustrated in Fig. 8. In Fig. 8, the first 1.1 s long segments of four room impulse responses are shown, the peak-to-noise ratio ranges between 42–50 dB.

C. Discussion

The peak-to-noise (P/N) ratios achieved in the two examples discussed previously can be further improved if an even longer reciprocal MLS pair can be used. The maximum achievable P/N ratio is restricted by the peak cross-correlation value of reciprocal MLS pairs listed in Table I. The PCCF between two reciprocal MLS pairs as shown in Fig. 1(b) is of a deterministic nature given the reciprocal MLS pair. Therefore, additional averages cannot significantly improve the P/N ratios if other kinds of random noise are in the measurement environment. For this reason, additional averages are not relevant for a P/N ratio improvement. Particularly, the technique employing reciprocal and preferred MLS pairs, as discussed in this paper is of practical significance especially for the simultaneous source channel measurement; a limited measurement time period is often critical in specific applications.

The separation of individual impulse responses from individual simultaneous sources is due to the excellent cross-correlation properties of reciprocal and preferred MLS pairs. The separation at the receiving ends, however, is not a big concern at all, since individual receiving channels inherently possess the separation ability. Increasing the number of receiving channels does not significantly influence the figure of achievable P/N ratio in the measured impulse responses, but increasing the number of simultaneous source channels does. Generally one more source channel leads to at least 3 dB degradation of achievable P/N ratio depending on the MLS selected.

More preferred MLS pairs need to be added to the simultaneous mode if a specific application requires more source channels given the limited measurement period. Some MLS-related sequences[22] can be employed if the number of available preferred MLS pairs cannot meet the need.

VI. CONCLUSION

The two-valued autocorrelation function of binary maximum-length sequences (MLS) has long been exploited for various applications, while considerably less attention has been given to the excellent cross-correlation property of binary MLS, particularly of reciprocal MLS. The cross-correlation function between a pair of reciprocal MLS exhibits relatively smaller values than the peak value of their autocorrelation function. It is this excellent property that makes the simultaneous dual-source measurements feasible. The measurement technique simultaneously obtains impulse responses of an acoustical system under test with two separate sound sources and one or several receivers with each of the reciprocal MLS pair exciting each of dual source channels. The impulse response measurements are based on a fast cross-correlation technique called the Fast MLS Transform. In this paper we have proposed the fast reciprocal MLS transform (FRMT) pair. The efficiency of the FRMT pair lies both in exploitation of the fast Hadamard transform algorithm and in the requirement of a single permutation matrix. In addition to the cross-correlation property, in this paper we have applied some fundamental properties of the binary MLS for a derivation of the permutation matrix from the MLS pair, including decimations, characteristic MLS, trace operations, and a trace orthogonal basis. The principle of the simultaneous dual source measurements is discussed and the efficient technique and potential acoustical applications have been demonstrated using exploratory experimental results. The technique proposed in this paper is especially of practical significance when impulse responses of acoustical systems with multiple sources and receivers have to be determined in a limited time period.

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