```
> restart:
> interface(imaginaryunit=j):
> with(Student[Calculus1]):
infolevel[Student[Calculus1]] := 1:
> f(x):=1/(x-sqrt(x-1));
```

$$f(x) := \frac{1}{x - \sqrt{x - 1}}$$

```
> Int(f(x),x);
```

$$\int \frac{1}{x - \sqrt{x - 1}}\, dx$$

```
> Hint(%);
Creating problem #1

Integrals of the form Int(R(X, sqrt(a*X+b), sqrt(c*X+d)),
X),where R is a rational function, can be reduced to integrals of
rational functions by means of the substitution (a*X+b) = u^2.
```

$$[change, x - 1 = u^2, u]$$

```
> Rule[%](%%);
Applying substitution x = 1+u^2, u = (x-1)^(1/2) with dx =
2*u*du, du = 1/2/(x-1)^(1/2)*dx
```

$$\int \frac{1}{x - \sqrt{x - 1}}\, dx = \int \frac{2\, u}{1 + u^2 - u}\, du$$

```
> Hint(%);
```

$$[constantmultiple]$$

```
> Rule[%](%%);
```

$$\int \frac{1}{x - \sqrt{x - 1}}\, dx = 2 \int \frac{u}{1 + u^2 - u}\, du$$

```
> Hint(%);
Rewrite the numerator in a form which contains the derivative of
the denominator
```

$$\left[rewrite, \frac{u}{1 + u^2 - u} = \frac{2\, u - 1}{2\,(1 + u^2 - u)} + \frac{1}{2\,(1 + u^2 - u)} \right]$$

```
> Rule[%](%%);
```

$$\int \frac{1}{x - \sqrt{x-1}}\, dx = 2 \int \frac{2\,u - 1}{2\,(1 + u^2 - u)} + \frac{1}{2\,(1 + u^2 - u)}\, du$$

```
> Hint(%);
```

$$[sum]$$

```
> Rule[%](%%);
```

$$\int \frac{1}{x - \sqrt{x-1}}\, dx = 2 \int \frac{2\,u - 1}{2\,(1 + u^2 - u)}\, du + 2 \int \frac{1}{2\,(1 + u^2 - u)}\, du$$

```
> Hint(%);
```

$$[constantmultiple]$$

```
> Rule[%](%%);
```

$$\int \frac{1}{x - \sqrt{x-1}}\, dx = \int \frac{2\,u - 1}{1 + u^2 - u}\, du + 2 \int \frac{1}{2\,(1 + u^2 - u)}\, du$$

```
> Hint(%);
Note that the derivative of 1+u^2-u is 2*u-1, so we can make a
change of variable.
```

$$[change, u1 = 1 + u^2 - u, u1]$$

```
> Rule[%](%%);
Applying substitution u = 1/2+1/2*(-3+4*u1)^(1/2), u1 = 1+u^2-u
with du = 1/(-3+4*u1)^(1/2)*du1, du1 = (2*u-1)*du
```

$$\int \frac{1}{x - \sqrt{x-1}}\, dx = \int \frac{1}{u1}\, du1 + 2 \int \frac{1}{2\,(1 + u^2 - u)}\, du$$

```
> Hint(%);
```

$$[power]$$

```
> Rule[%](%%);
```

$$\int \frac{1}{x - \sqrt{x-1}}\, dx = \ln(u1) + 2 \int \frac{1}{2\,(1 + u^2 - u)}\, du$$

```
> Hint(%);
```

$$[revert]$$

```
> Rule[%](%%);
Reverting substitution using u1 = 1+u^2-u
```

$$\int \frac{1}{x - \sqrt{x-1}}\,dx = \ln(1 + u^2 - u) + 2\int \frac{1}{2\,(1 + u^2 - u)}\,du$$

```
> Hint(%);
```

$$[constantmultiple]$$

```
> Rule[%](%%);
```

$$\int \frac{1}{x - \sqrt{x-1}}\,dx = \ln(1 + u^2 - u) + \int \frac{1}{1 + u^2 - u}\,du$$

```
> Hint(%);
Complete the square and make a change of variable.
```

$$\left[change, u1 = u - \frac{1}{2}, u1 \right]$$

```
> Rule[%](%%);
Applying substitution u = u1+1/2 with du=du1
```

$$\int \frac{1}{x - \sqrt{x-1}}\,dx = \ln(1 + u^2 - u) + \int \frac{4}{4\,u1^2 + 3}\,du1$$

```
> Hint(%);
```

$$[constantmultiple]$$

```
> Rule[%](%%);
```

$$\int \frac{1}{x - \sqrt{x-1}}\,dx = \ln(1 + u^2 - u) + 4\int \frac{1}{4\,u1^2 + 3}\,du1$$

```
> Hint(%);
Integrals involving expressions of the form (x^2+a^2)^n or
sqrt(x^2+a^2)^n can often be simplified using the substitution x
= a*tan(u).
```

$$\left[change, u1 = \frac{1}{2}\sqrt{3}\,\tan(u2), u2 \right]$$

```
> Rule[%](%%);
Applying substitution u1 = 1/2*3^(1/2)*tan(u2), u2 =
arctan(2/3*u1*3^(1/2)) with du1 = 1/2*3^(1/2)*(1+tan(u2)^2)*du2,
du2 = 2/3*3^(1/2)/(4/3*u1^2+1)*du1
```

$$\int \frac{1}{x - \sqrt{x-1}}\,dx = \ln(1 + u^2 - u) + 4\int \frac{1}{6}\sqrt{3}\,du2$$

```
> Hint(%);
```

$$[constant]$$

```
> Rule[%](%%);
```

$$\int \frac{1}{x - \sqrt{x-1}}\, dx = \ln(1 + u^2 - u) + \frac{2}{3}\sqrt{3}\ u2$$

```
> Hint(%);
```

$$[revert]$$

```
> Rule[%](%%);
Reverting substitution using u2 = arctan(2/3*u1*3^(1/2))
```

$$\int \frac{1}{x - \sqrt{x-1}}\, dx = \ln(1 + u^2 - u) + \frac{2}{3}\sqrt{3}\ \arctan\left(\frac{2}{3}\, u1 \sqrt{3}\right)$$

```
> Hint(%);
```

$$[revert]$$

```
> Rule[%](%%);
Reverting substitution using u1 = u-1/2
```

$$\int \frac{1}{x - \sqrt{x-1}}\, dx = \ln(1 + u^2 - u) + \frac{2}{3}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\ (2\, u - 1)\right)$$

```
> Hint(%);
```

$$[revert]$$

```
> Rule[%](%%);
Reverting substitution using u = (x-1)^(1/2)
```

$$\int \frac{1}{x - \sqrt{x-1}}\, dx = \ln(x - \sqrt{x-1}) + \frac{2}{3}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\ (2\sqrt{x-1} - 1)\right)$$

```
> Hint(%);
This problem is complete
```

$$[]$$

```
> ShowSteps();
```

$$\int \frac{1}{x - \sqrt{x-1}}\, dx = \int \frac{2\, u}{1 + u^2 - u}\, du$$

$$= 2 \int \frac{u}{1 + u^2 - u} \, du$$

$$= 2 \int \frac{2\,u - 1}{2\,(1 + u^2 - u)} + \frac{1}{2\,(1 + u^2 - u)} \, du$$

$$= 2 \int \frac{2\,u - 1}{2\,(1 + u^2 - u)} \, du + 2 \int \frac{1}{2\,(1 + u^2 - u)} \, du$$

$$= \int \frac{2\,u - 1}{1 + u^2 - u} \, du + 2 \int \frac{1}{2\,(1 + u^2 - u)} \, du$$

$$= \int \frac{1}{u1} \, du1 + 2 \int \frac{1}{2\,(1 + u^2 - u)} \, du$$

$$= \ln(u1) + 2 \int \frac{1}{2\,(1 + u^2 - u)} \, du$$

$$= \ln(1 + u^2 - u) + 2 \int \frac{1}{2\,(1 + u^2 - u)} \, du$$

$$= \ln(1 + u^2 - u) + \int \frac{1}{1 + u^2 - u} \, du$$

$$= \ln(1 + u^2 - u) + \int \frac{4}{4\,u1^2 + 3} \, du1$$

$$= \ln(1 + u^2 - u) + 4 \int \frac{1}{4\,u1^2 + 3} \, du1$$

$$= \ln(1 + u^2 - u) + 4 \int \frac{1}{6}\sqrt{3}\ du2$$

$$= \ln(1 + u^2 - u) + \frac{2}{3}\sqrt{3}\ u2$$

$$= \ln(1 + u^2 - u) + \frac{2}{3}\sqrt{3}\ \arctan\left(\frac{2}{3}\,u1\sqrt{3}\right)$$

$$= \ln(1 + u^2 - u) + \frac{2}{3}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\ (2\,u - 1)\right)$$

$$= \ln(x - \sqrt{x - 1}) + \frac{2}{3}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\ (2\sqrt{x - 1} - 1)\right)$$

>