5.13 Three-pole, single amplifier filter

A more important set of instruction books will never be found by human beings. When finally interpreted the genetic messages encoded within our DNA will provide the ultimate answers to the chemical underpinnings of human existence.

James Watson

We consider here the design of a three-pole filter using only a single amplifier. This would require us to solve third order equations which makes the calculation somewhat complex to carry out by hand, but the use of mathematical software packages now makes this relatively simple (Brokaw 1970: Rutschow 1998). Most active filters of this form use unity, or very low, overall gain. This circuit is somewhat unusual in that we have found it to be usable at very high gain; the matter of allowable gain will be discussed later. The circuit is shown in Fig. 5.13.1.

The general transfer function for a three-pole low-pass filter is given by:

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{A_3 s^3 + A_2 s^2 + A_1 s + A_0} \tag{5.13.1}$$

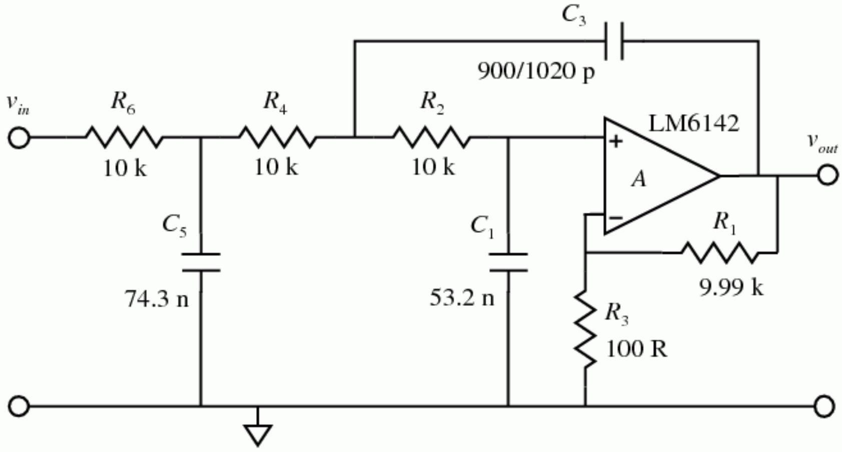

Fig. 5.13.1 Three-pole single amplifier low-pass filter.

Table 5.13.1 *Three-pole single amplifier filter parameter values for various responses (after Rutschow 1998)*

Filter type	Features	A_1	A_2	A_3	Corner attenuation (dB)
Butterworth	Maximally flat passband	2	2	1	3
Chebyshev −1 dB ripple	Equal 1 dB passband ripples and very rapid cut-off	2.52071	2.01164	2.03537	1
−3 dB ripple	Equal 3 dB passband ripples and very rapid cut-off	3.70466	2.38334	3.99058	3
Optimal (Papoulis)	Rapid cut-off and monotonic in passband	2.35529	2.27036	1.7331	3
Bessel (Thomson)	Approximates Gaussian response. Minimizes phase delay distortion	1	0.4	0.06667	0.84
Paynter	Excellent time domain response. Minimal overshoot	3.2	4	3.2	10.4

The gain can be normalized to unity by dividing through by A_0. A difficulty in determining the relationships between the components arises because there are only three equations for six unknowns. For a corner frequency of $f_0 = \omega_0/2\pi$ and gain K these are:

$$A_1 = [(1-K)(R_6+R_4)C_3 + R_6C_5 + (R_2+R_4+R_6)C_1]\omega_0$$
$$A_2 = [(1-K)R_6R_4C_5C_3 + R_6(R_2+R_4)C_1C_5 + R_2(R_4+R_6)C_1C_3]\omega_0^2$$
$$A_3 = R_6R_4R_2C_5C_3C_1\omega_0^3$$
$$K = 1 + \frac{R_1}{R_3} \tag{5.13.2}$$

Various types of response can be obtained according to the relationships between the A's as shown in Table 5.13.1 (the A values are normalized to $A_0 = 1$; Kuo 1966). To find appropriate values it is necessary to select three and then to seek the best values for the other three. This can be carried out with the aid of Mathcad using the *Given* and *Find* commands, with the capacitors as variables, setting say the three resistor values (set them to the same value to start: as in most circumstances in electronics, if in doubt use 10 k! – as with the three bears, it is not too big, it is not too small and may be just right) and choosing the appropriate values of the A's for the desired filter type. Some nominal capacitor values also need to be entered to provide a starting point for the search. This gives the calculated capacitor values. The nearest standard values for the capacitors are then set and the resistor values determined instead in the same way. The nearest standard values are then used. To show the frequency response the circuit is transferred to PSpice to see if it is acceptable. It should be noted that, as with all circuits in which positive and negative feedback are counterbalanced, the response is somewhat sensitive to gain K. It is difficult to provide an expression for the limit since all the components themselves depend on the value of K chosen. Mathematically, the simplest transfer function of the filter is obtained when all the R's and all the C's are equal. Equation (5.13.1) then becomes:

$$\frac{V_{out}}{V_{in}}=\frac{1}{R^3C^3s^3+R^2C^2(5-K)s^2+2RC(3-K)s+1},\quad \text{for}\quad A_0=1 \tag{5.13.3}$$

and we may examine the stability using the Routh rules given in Section 1.12. The values of the three variables are (dividing through by the coefficient of s^3):

$$\alpha=\frac{(5-K)}{RC},\qquad \beta=\frac{2(3-K)}{R^2C^2},\qquad \gamma=\frac{1}{R^3C^3} \tag{5.13.4}$$

so K must be less than 3 otherwise β will change sign and by Rule 1 the system will be unstable. If $K<3$ so that the first part of Rule 3 is fulfilled, then applying the second part for the *limit of equality*:

$$\beta=\frac{\gamma}{\alpha}\qquad \text{gives}\qquad \frac{2(3-K)}{R^2C^2}=\frac{1}{R^3C^3}\frac{RC}{(5-K)}$$
$$\text{or}\quad 2K^2-16K+29=0\qquad \text{so}\qquad K=5.225\quad \text{or}\quad 2.775 \tag{5.13.5}$$

using the standard formula for quadratics (Section 1.10). The higher value has already been eliminated and thus the maximum value for K is 2.775 rather than 3. This can now be examined using PSpice. The frequency response will show a large peak around the corner frequency and a transient run, using a short pulse input to nudge the system, will show oscillations. It is found that the K limit is accurate; a gain of 2.8 gives growing oscillations, while a gain of 2.75 gives decaying oscillation. A gain of 2.775 gives effectively constant oscillation. It is instructive to apply the T technique (Section 5.14) by applying a voltage generator at the (+) input of the amplifier (this will mean that the open-loop response will follow T_v). If you examine the loop gain at zero phase shift you will find it very close to unity. You should also note the change of phase of the signal at (+).

This limit of $K\approx3$ is not as restrictive as it appears. The result arises from the arbitrary choice of the components without consideration of the resulting response, but it at least reminds us that the system is capable of oscillation. The Butterworth configuration has been investigated as a function of K up to 1000. It is possible to achieve stability even at this very high gain but the system is now very sensitive to the value of C_3 and the frequency response is considerably rounded (suitable values to try for a LM6142 amplifier with 1 kHz cut-off are: $R_2 = R_4 = R_6 =$ 10 k, $C_1 = 118.9$ n, $C_3 = 235$ p, $C_5 = 101.5$ n with the feedback resistors 9.99 k and 10R). Increasing C_3 to 240 p results in oscillation while reduction to 200 p gives better transient response. What has not been allowed for in the design equations is the amplifier open-loop response which will become more significant as gain or cut-off frequency increases. It is somewhat astonishing that such high gains are feasible but you need to investigate the performance carefully. In carrying out the calculations it is probably best to start with low gain and increase the gain in limited steps, feeding back the new values as better approximations for the next step to avoid convergence failure.

As an example Fig. 5.13.2(*a*) shows the Butterworth frequency response for a gain of one hundred and a design corner of 1 kHz for two values of C_3. The input signal was 1 mV.

The cursors (set two decades apart) indicate a slope of about 60 dB/dec as expected for a third order response. If the frequency response appears satisfactory then the system should also be tested with a pulse input to examine the transient response. Figure 5.13.2(*b*) shows the response for a 1 mV, 10 μs risetime pulse with the same two values of C_3; the decrease in overshoot is at the expense of risetime. For better pulse response the Paynter configuration gives a fast risetime with only a small overshoot. Increasing C_1 will remove the overshoot at the expense of increased risetime. Even if the gain is low it is as well to use an amplifier with a good gain–bandwidth product so that the amplifier roll-off does not significantly affect the response.

For intermediate response between Butterworth and Bessel (Thomson) reference may be made to Al-Nasser (1972), Melsheimer (1967) or Van Valkenburg (1982). These provide pole locations for variation between Butterworth ($m = 0$) and Thomson ($m = 1$) but some sums are required to derive the equivalent A coefficients. If the real pole is $|a|$ and the complex poles are $|b \pm jc|$ (i.e. ignore the minus signs in the tables), then the denominator polynomial is given by:

$$s^3 + s^2(a + 2b) + s(2ab + b^2 + c^2) + a(b^2 + c^2)$$

$$\text{so} \quad A_3 = \frac{1}{a(b^2 + c^2)}, \quad A_2 = \frac{(a + 2b)}{a(b^2 + c^2)}, \quad A_1 = \frac{(2ab + b^2 + c^2)}{a(b^2 + c^2)}, \quad A_0 = 1 \qquad (5.13.6)$$

The references also give some advice on the *GB* product required for the amplifier. Papoulis (1958) deals with the optimal filter.

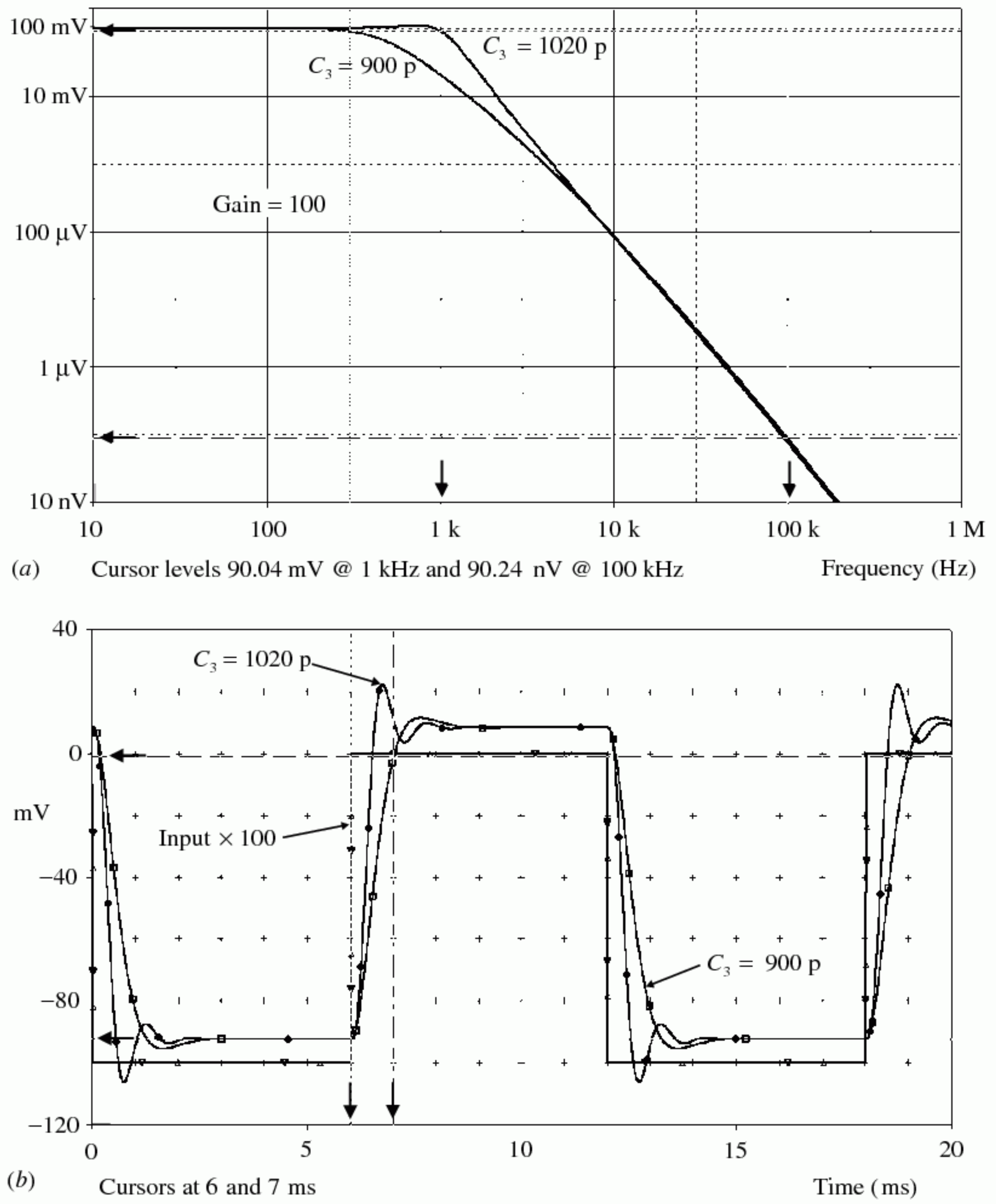

Fig. 5.13.2 (*a*) Frequency response of circuit of Fig. 5.13.1 with values as shown. (*b*) Transient response for the circuit (the outputs have been offset for clarity).

SPICE simulation circuits

Fig. 5.13.2(*a*)	3psalpf5.SCH Equal *R*'s and equal *C*'s
Fig. 5.13.2(*a*)	3psalpf7.SCH Gain = 100, values as 5.13.1
Fig. 5.13.2(*b*)	3psalpf6.SCH Gain = 100, pulse response
Paynter response	3psalpf4.SCH

References and additional sources 5.13

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