

$$\Phi_X(j\omega) = e^{-\omega^2 \sigma_X^2 / 2 + j\omega m_X}. \quad (210)$$

The moment theorem yields for orders $n = 1, \dots, 4$ (see Eqs.135 to 138)

$$m_X^{(1)} = m_X, \quad (211)$$

$$m_X^{(2)} = \sigma_X^2 + m_X^2, \quad (212)$$

$$m_X^{(3)} = 3\sigma_X^2 m_X + m_X^3, \quad (213)$$

$$m_X^{(4)} = 3\sigma_X^4 + 6\sigma_X^2 m_X^2 + m_X^4. \quad (214)$$

The **cumulants** were obtained from the cumulant generating function (see Eq.126)

$$\Psi_X(s) = \ln \Phi_X(s) = s^2 \sigma_X^2 / 2 + s m_X \quad (215)$$

as (cf. Eqs.142 to 145):

$$\lambda_X^{(0)} = s^2 \sigma_X^2 / 2 + s m_X \Big|_{s=0} = 0, \quad (216)$$

$$\lambda_X^{(1)} = s \sigma_X^2 + m_X \Big|_{s=0} = m_X, \quad (217)$$



$$\lambda_X^{(2)} = \sigma_X^2 \Big|_{s=0} = \sigma_X^2, \quad (218)$$

$$\lambda_X^{(n>2)} = 0. \quad (219)$$

Definition 2.37. Two jointly normally distributed RVs. Two RVs X, Y are **jointly normal** or **jointly Gaussian**, denoted by $\mathcal{N}(m_X, m_Y, \sigma_X, \sigma_Y, c_{XY})$, if their joint pdf reads

$$f_{XY}(x, y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-c_{XY}^2}} \cdot \exp \left\{ -\frac{1}{2(1-c_{XY}^2)} \left[\frac{(x-m_X)^2}{\sigma_X^2} - 2c_{XY} \cdot \frac{(x-m_X)(y-m_Y)}{\sigma_X\sigma_Y} + \frac{(y-m_Y)^2}{\sigma_Y^2} \right] \right\} \quad (220)$$

where c_{XY} with $|c_{XY}| < 1$ denotes the correlation coefficient (see Eq.162).

