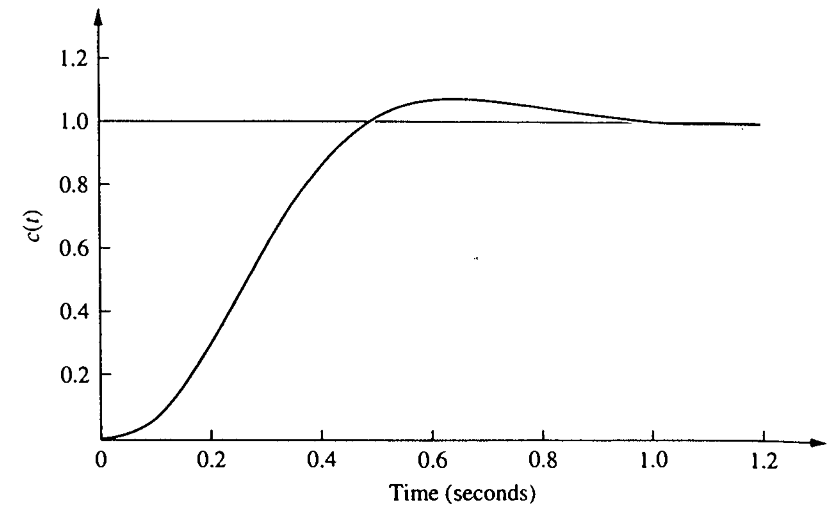


Figure 9.57 Step response simulation for Example 9.8

9.6 Physical Realization of Compensation

In this chapter we derived compensation to improve transient response and steady-state error in feedback control systems. Transfer functions of compensators used in cascade with the plant or in the feedback path were derived. These compensators were defined by their pole-zero configuration. They were either active PI, PD, or PID controllers or passive lag, lead, or lag-lead compensators. In this section we show how to implement the active controllers and the passive compensators.

Active-Circuit Realization

In Chapter 2 we derived

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} \tag{9.44}$$

as the transfer function of an inverting operational amplifier whose configuration is repeated here in Figure 9.58. By judicious choice of $Z_1(s)$ and $Z_2(s)$ this circuit can be used as a building block to implement the compensators and controllers, such as PID controllers, discussed in this chapter. Table 9.10 summarizes the realization of PI, PD, and PID controllers as well as lag, lead, and lag-lead compensators using

Figure 9.58 Operational amplifier configured for transfer function realization

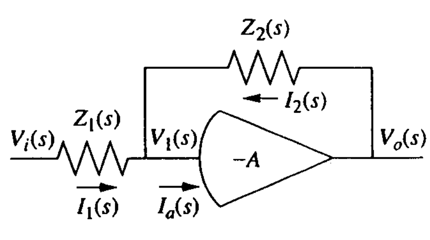
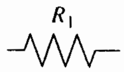
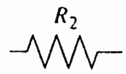
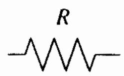
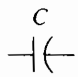
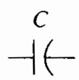
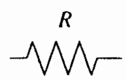
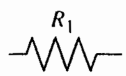
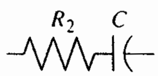
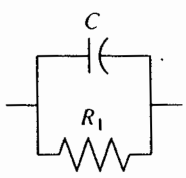
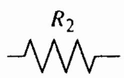
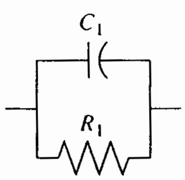
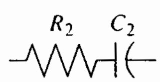
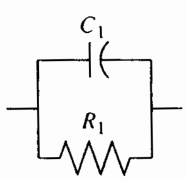
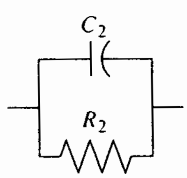
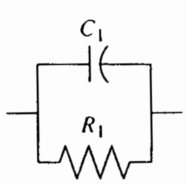
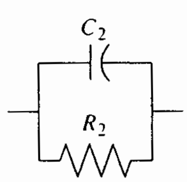


Table 9.10 Active realization of controllers and compensators, using an operational amplifier

Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\frac{Z_2(s)}{Z_1(s)}$
Gain			$-\frac{R_2}{R_1}$
Integration			$-\frac{\frac{1}{RC}}{s}$
Differentiation			$-RCs$
PI controller			$-\frac{R_2}{R_1}\frac{\left(s+\frac{1}{R_2C}\right)}{s}$
PD controller			$-R_2C\left(s+\frac{1}{R_1C}\right)$
PID controller			$-\left[\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)+R_2C_1s+\frac{\frac{1}{R_1C_2}}{s}\right]$
Lag compensation			$-\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ where $R_2C_2 > R_1C_1$
Lead compensation			$-\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ where $R_1C_1 > R_2C_2$

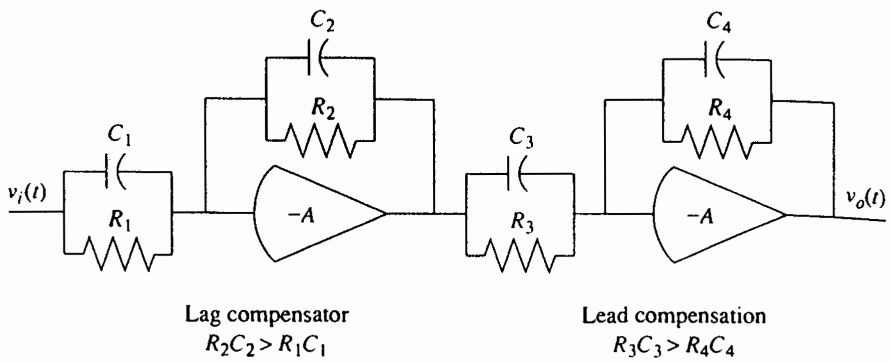


Figure 9.59 Lag-lead compensator implemented with operational amplifiers

operational amplifiers. You can verify the table by using the methods of Chapter 2 to find the impedances.

Other compensators can be realized by cascading compensators shown in the table. For example, a lag-lead compensator can be formed by cascading the lag compensator with the lead compensator, as shown in Figure 9.59. As an example, let us implement one of the controllers we designed earlier in the chapter.

Example 9.9

Implementing a PID controller

Problem Implement the PID controller of Example 9.5.

Solution The transfer function of the PID controller is $G_c(s) = (s + 55.909)(s + 0.5)/s$, which can be put in the form, $G_c(s) = s + 56.409 + (27.954/s)$. From the PID controller in Table 9.10, $[(R_2/R_1) + (C_1/C_2)] = 56.409$, $R_2C_1 = 1$, and $1/R_1C_2 = 27.954$.

Since there are four unknowns and three equations, we arbitrarily select a practical value for one of the elements. Selecting $C_2 = 0.1\ \mu$F, the remaining values are found to be $R_1 = 357.731$ kΩ, $R_2 = 178.862$ kΩ, and $C_1 = 5.591\ \mu$F.

The complete circuit is shown in Figure 9.60, where the circuit element values have been rounded off.

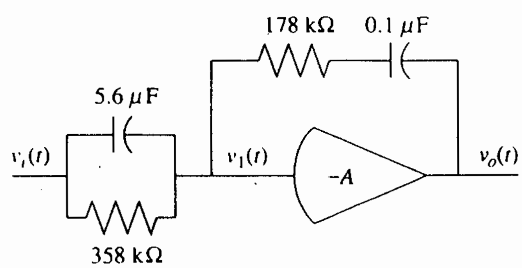


Figure 9.60 PID controller

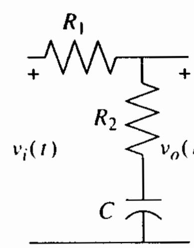
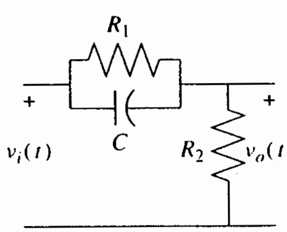
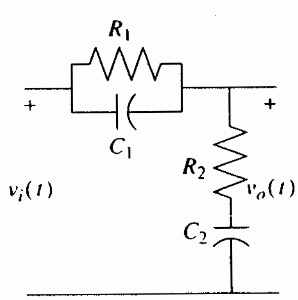
Passive-Circuit Realization

Lag, lead, and lag-lead compensators can also be implemented with passive networks. Table 9.11 summarizes the networks and their transfer functions. The transfer functions can be derived with the methods of Chapter 2.

The lag-lead transfer function can be put in the following form:

$$G_c(s) = \frac{\left(s + \frac{1}{T_1}\right)\left(s + \frac{1}{T_2}\right)}{\left(s + \frac{1}{\alpha T_1}\right)\left(s + \frac{\alpha}{T_2}\right)} \tag{9.45}$$

Table 9.11 Passive realization of compensators

Function	Network	Transfer function, $\frac{V_o(s)}{V_i(s)}$
Lag compensation		$\frac{R_2}{R_1 + R_2}\,\frac{s + \frac{1}{R_2 C}}{s + \frac{1}{(R_1 + R_2)C}}$
Lead compensation		$\frac{s + \frac{1}{R_1 C}}{s + \frac{1}{R_1 C} + \frac{1}{R_2 C}}$
Lag-lead compensation		$\frac{\left(s + \frac{1}{R_1 C_1}\right)\left(s + \frac{1}{R_2 C_2}\right)}{s^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1}\right)s + \frac{1}{R_1 R_2 C_1 C_2}}$

where $\alpha < 1$. Thus the terms with T_1 form the lead compensator, and the terms with T_2 form the lag compensator. Equation (9.45) shows a restriction inherent in using this passive realization. We see that the ratio of the lead compensator zero to the lead compensator pole must be the same as the ratio of the lag compensator pole to the lag compensator zero. In Chapter 11 we design a lag-lead compensator with this restriction.

A lag-lead compensator without this restriction can be realized with an active network as previously shown or with passive networks by cascading the lead and lag networks shown in Table 9.11. Remember, though, that the two networks must be isolated to ensure that one network does not load the other. If the networks load each other, the transfer function will not be the product of the individual transfer functions. A possible realization using the passive networks uses an operational amplifier to provide isolation. The circuit is shown in Figure 9.61. Example 9.10 demonstrates the design of a passive compensator.

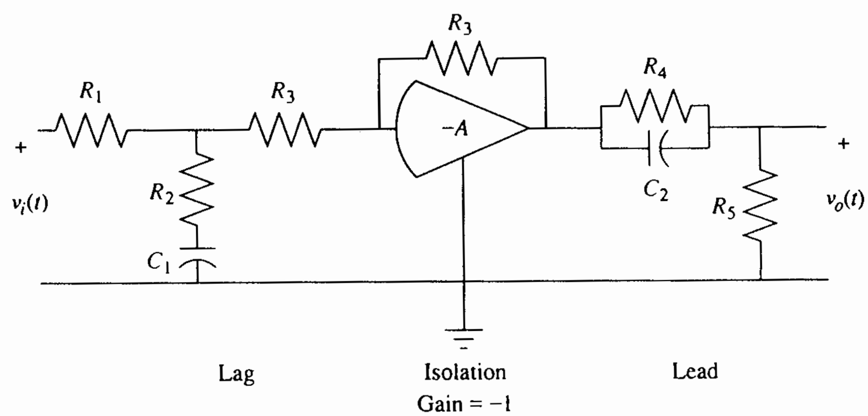


Figure 9.61 Lag-lead compensator implemented with cascaded lag and lead networks with isolation

Example 9.10

Realizing a lead compensator

Problem Realize the lead compensator designed in Example 9.4 (Compensator *b*).

Solution The transfer function of the lead compensator is $G_c(s) = (s + 4)/(s + 20.089)$. Thus, from the transfer function of a lead network shown in Table 9.11, $1/R_1C = 4$, and $(1/R_1C) + (1/R_2C) = 20.089$. Hence $R_1C = 0.25$, and $R_2C = 0.0622$. Since there are three network elements and two equations, we may select one of the element values arbitrarily. Letting $C = 1\ \mu F$, $R_1 = 250\ k\Omega$ and $R_2 = 62.2\ k\Omega$.