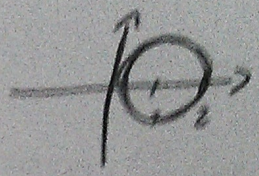
$a = 1/2$

$$H(e^{j\omega}) = \left(e^{j\omega}\left(1-\frac{1}{2}\right)+\frac{1}{2}\right)/e^{j\omega}$$

$$= (e^{j\omega}+1)/2\cdot e^{j\omega}$$

$$= \frac{1}{2}\left(1+e^{-j\omega}\right)$$



$$\angle H = \angle\left(1+e^{-j\omega}\right) \Rightarrow$$

$$= \angle(\cos(-\omega)+1 \;/\; \sin(-\omega))$$

$$= \angle(\cos\omega+1 \;-\; j\sin\omega)$$

$$\angle H = \operatorname{atan}\left(\frac{-\sin\omega}{\cos\omega+1}\right)$$

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① $\dfrac{\delta\left(\operatorname{atan}\left(\frac{-\sin\omega}{\cos\omega+1}\right)\right)}{\delta\omega} \overset{?!}{=} \text{const} \;?\; = -\frac{1}{2}$

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② $\left((\cos\omega+1)^{-1}\right)' = -\frac{1}{(\cos\omega+1)^2}\,(-\sin\omega)$

$$= \frac{\sin\omega}{(\cos\omega+1)^2}$$

---

③ $(\sin\omega\cdot(\cos\omega+1)^{-1})'$

$$= \frac{\cos\omega}{\cos\omega+1} + \frac{\sin^2\omega}{(\cos\omega+1)^2} = \frac{\cos\omega(\cos\omega+1)+\sin^2\omega}{(\cos\omega+1)^2}$$

$$= \frac{1}{\cos\omega+1}$$

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④ $(\operatorname{atan} x)' = \frac{1}{1+x^2}$

---

② ③ ④ in ①

$$\operatorname{atan}\left(\frac{-\sin\omega}{\cos\omega+1}\right)'$$

$$= \frac{1}{1 + \left(\frac{-\sin u}{\cos u + 1}\right)^2} \cdot \frac{-1}{(\cos u + 1)}$$

$$= \frac{-1}{\cos u + 1 + \frac{\sin^2 u}{(\cos u + 1)}}$$

$$= \frac{-(\cos u + 1)}{(\cos u + 1)^2 + \sin^2 u} = -\frac{\cos u + 1}{\cos^2 u + 2\cos u + \sin^2 u}$$

$$= -\frac{\cos u + 1}{2\cos u + 2} = -\frac{1}{2}$$ qed