A Guide to Calculate Convection Coefficients for Thermal Problems
Application Note

Keywords: Thermal analysis, convection coefficients, computational fluid dynamics, free convection, forced convection.

Abstract: The present application note is a guide that can be used to calculate thermal convection coefficients. As such it approaches the topic of computational fluid dynamics (CFD) since the problem of calculating convection coefficients is situated at the intersection between CFD and Thermal analysis. Some basic presentation of the theoretical background is also included for more clarity of the approach of the subject. A few examples of the computational process are also included and deal with practical cases of free convection as well as forced convection.

Procedure to calculate convection coefficients

1. Introduction

As widely known the specification of convection coefficients is a necessity in thermal applications when cooling of surfaces in contact with fluids (liquids and/or gases) occurs and a thermal convection mechanism takes place.

At its core the problem is in fact one in which the thermal aspect is strongly coupled with the fluid flow aspect: temperature distribution influences the fluid flow characteristic quantities while the fluid flow parameters influence the temperature distribution. It can be said that in such a scenario convection coefficients –if necessary at all- can be obtained as one of the results of a coupled Computational Fluid Dynamics (CFD) – Thermal analysis. While the full analysis of the coupled problem is quite complicated, there are ways to simplify the computational task and produce results for the convection coefficients that have acceptable accuracy. One of the most widely spread is the method using dimensionless parameters. This method is quite easy to use, however it has the disadvantage that it doesn't allow an understanding of underlying physics of this complex phenomenon. This is why in the following paragraphs we'll also discuss the basics of the physical mechanisms of heat convection since a minimal understanding on the physical background is necessary. Only after that we'll present the "mechanics" of calculating the convection coefficients based on formulas and "numbers".

When it comes to convection, two main cases need to be considered. The case *of free (natural) convection* and the case of *forced convection*.

The essential components of heat transfer by convection mechanisms are given in Newton's law of cooling:

$$Q = h \cdot A \cdot \left(T_w - T_\infty\right) \qquad (1)$$

where Q is the rate of heat (power) transferred between the exposed surface A of the wall and the fluid. T_w is the temperature of the wall, T_∞ is the temperature of the free stream of fluid, h is the convection coefficient (also called film coefficient), the quantity we want to calculate.

An observation is in order regarding (1): the equation should be seen more as a definition of the convection coefficient h rather than a law of heat transfer by convection. Indeed, equation (1) doesn't explain anything about the convection mechanism, which is a rather complicated one. So the simplicity of (1) is misleading and in sharp contrast with what we know about the complexity of the convection phenomenon. All that complexity is revealed only when we investigate the ways to find the dependencies of h –the convection coefficient- on many factors. We find that h is a complicated function of the fluid flow, thermal properties of the fluid, the geometry and orientation in space of the system that exhibits convection. Moreover, the convection coefficient is not uniform on the entire surface and it also depends on the location where the temperature of the fluid is evaluated. In most cases it is therefore convenient and practical to use average values for the convection coefficients.

2. Quantities and numbers frequently used in convection coefficient calculations

The occurrence of turbulent flow is usually correlated with Rayleigh number, which is simply the product of the Grashof and Prandtl numbers:

$$Ra = Gr \cdot \mathrm{Pr} \tag{2}$$

Grashof number is given by the expression in (3)

$$Gr = \frac{L^3 \cdot g \cdot \rho^2 \cdot \beta \cdot (T_w - T_\infty)}{\mu^2} = \frac{L^3 \cdot g \cdot \beta \cdot (T_w - T_\infty)}{\nu^2} \tag{3}$$

where L [m] is the characteristic length, g [m / s^2] is gravity, ρ [Kg /m^3] is the density of the fluid, β [1/K] is the thermal expansion coefficient , μ [Kg / m s] is the dynamic viscosity, ν [m^2 / s] is the kinematic viscosity.

All fluid characteristics (such as density, viscosity, etc) are evaluated at film temperature,

$$T_f = \frac{T_w + T_\infty}{2} \tag{4}$$

For perfect (ideal) gases $\beta = \frac{1}{T_f}$, while for liquids and non-ideal gases the expansion coefficient must be obtained from appropriate property tables.

Prandtl number is the ratio of two molecular transport properties, the kinematic viscosity ν which affects the velocity profile and the thermal diffusivity α, which affects the temperature profile.

$$\alpha = \frac{k}{\rho \cdot c_p}$$

$$\nu = \frac{\mu}{\rho}$$

$$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{c_p \cdot \mu}{k}$$

where k [W/m K] is the fluid thermal conductivity, c_p [J/Kg K] is the fluid specific heat.

Nusselt number is used directly to evaluate the convection coefficient according to (5):

$$\overline{Nu} = \frac{\overline{h} \cdot L}{k} \qquad (5)$$

In equation (5) the bar above quantities signifies an average value.

A few comments are necessary regarding the characteristic length. For a vertical wall the characteristic length is the height H of the wall (for free convection), for horizontal cylinder or for a sphere the characteristic length is the diameter D.

3. The case of free convection

Free convection occurs when an object is immersed in a fluid at a different temperature. It is considered that the fluid is in a quiescent state. In this situation there is an energy exchange between the fluid and the object. From a physics perspective the heat exchange is due to buoyancy forces caused by density gradients developed in the body of the fluid. As a result free convection currents are produced that facilitate the heat transfer between the object and the fluid. The body force responsible for the free convection currents is – as a rule - of gravitational nature (occasionally it may be centrifugal in rotating machines). Because gravity plays an important role in the convection phenomenon, the orientation in space of the respective walls matters for the calculation process of the respective convection coefficients.

Free convection originates from a thermal instability: warmer air moves upward while the cooler, heavier air moves downward. A different type of instabilities may also arise: hydrodynamic instabilities that relate to a transition between a laminar flow to a turbulent flow. The laminar or turbulent character of the fluid flow impact on the heat exchange mechanism between object walls and fluids. Therefore we have to have a way to characterize the hydrodynamic character of the flow in order to choose the applicable formulas for the calculation of the convection coefficients. Since the mechanism of convection is strongly dependent on both the fluid flow pattern and on the temperature distribution in the vicinity of the wall we'll take a look at the respective profiles.

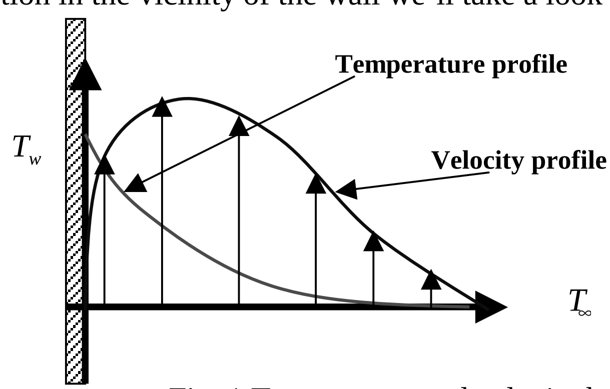

Fig. 1 Temperature and velocity boundary layers

Due to the viscosity of the fluid, a very thin layer (a few molecular mean free path thick) doesn't move relative to the wall. This makes the velocity to increase from zero at the wall to a maximum value and then decrease back to zero where the uniform temperature of the fluid (air) is reached. The temperature decreases from the value at the wall T_w to room temperature T_∞ in the same distance from the wall. It is now clear that the temperature and velocity distributions are strongly interrelated. Also the distance from the wall in which they are related is the same since due to the uniformity of the temperature (ambient room temperature) the differences in density cease to exist. As a direct consequence the buoyancy in the volume of the fluid disappears too.

Another observation that can be made is that due to the motionless of the air in the immediate vicinity of the wall, the heat transfer through this layer is by thermal conduction only. The actual convection mechanism is only active away from the wall. So in what we usually call convection, both thermal conduction and convection are present and interdependent. Considering a constant temperature wall, there is a relatively small temperature increase as the air moves upward. The increase of temperature in this boundary layer near the wall causes the heat transfer rate to decrease in the upward direction. For a sufficiently high wall the flow pattern can change from a laminar flow to a turbulent flow. This is another reason why convection is such a complicated phenomenon and quite difficult to treat with very high accuracy. The calculation of an average convection heat transfer coefficient is an approximation but as a rule a satisfactory one in most practical applications where convection is present.

For most engineering calculations the convection coefficient is obtained from a relation of the form:

$$\overline{Nu} = \frac{\overline{h} \cdot L}{k} = C \cdot Ra^n \tag{6}$$

where Ra is the Rayleigh number (2), C and n are coefficients. Typically n = 1/4 for laminar flow and n = 1/3 for turbulent flow. For turbulent flow Ra > 10^9, for laminar flow Ra < 10^9.

3.1. Vertical walls

Expressions of the form (6) are used for vertical isothermal walls and are plotted in Fig. 2. All applicable properties of the fluid are evaluated at film temperature T_f . For Rayleigh numbers less than 10^4, the Nusselt number should be obtained directly from Fig. 2 rather than using (6).

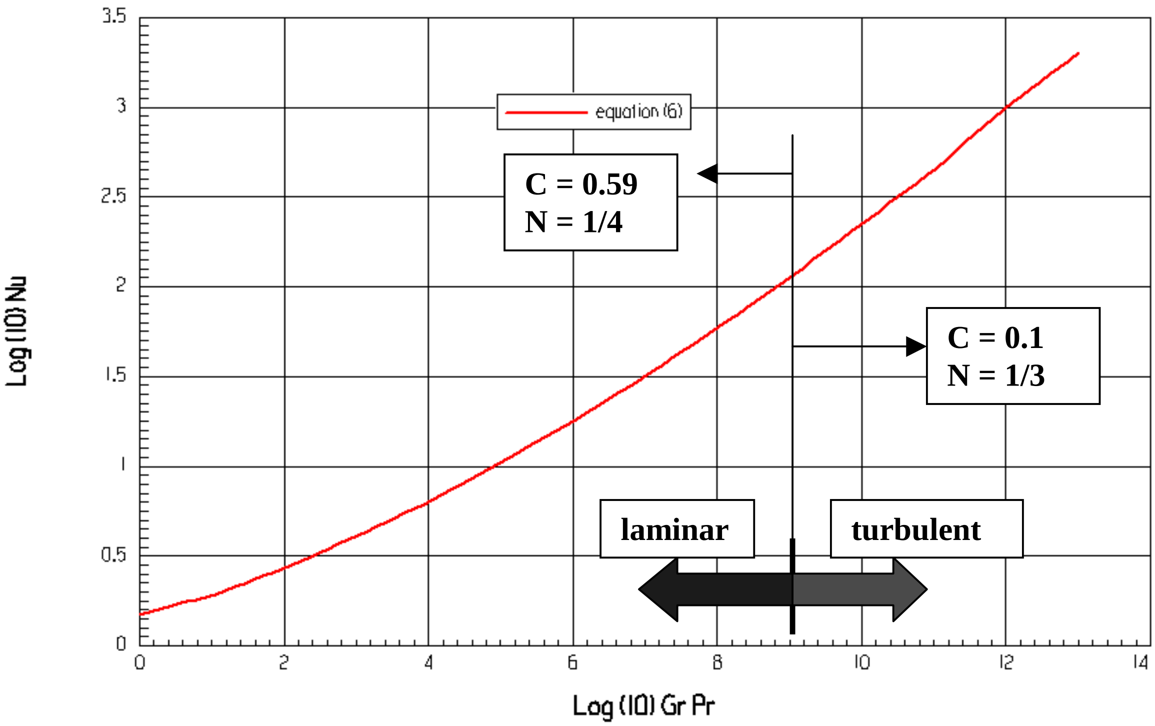

Fig. 2 Nusselt number for natural convection (vertical wall)

Strictly speaking equations of form (6) are applicable for isothermal walls. There may be situations where the wall may exhibit a uniform heat flux over the surface. In this case, the temperature over the surface of the wall is not necessarily constant. The film temperature is to be calculated with the temperature of the wall measured at the midpoint of the wall.

An alternative formula that can be used for the entire range of Rayleigh numbers is presented in (7a):

$$\overline{Nu} = \left\{ 0.825 + \frac{0.387 \cdot Ra^{1/6}}{[1 + (0.492/\Pr)^{9/16}]^{8/27}} \right\}^2 \qquad (7a)$$

For the case of laminar flow equation (7a) is applicable, however a slightly better accuracy is obtained from (7b):

$$\overline{Nu} = 0.68 + \frac{0.67 \cdot Ra^{1/6}}{[1 + (0.492/\Pr)^{9/16}]^{4/9}} \qquad Ra \le 10^9 \qquad (7b)$$

3.2 Inclined surfaces

The mechanics of the heat transfer is now different from the case with vertical walls. For an inclined wall the buoyancy force has a component normal as well as parallel to the respective surface. Therefore there will be a reduction in fluid velocity in a direction parallel to the plate. This reduction however doesn't necessarily translate in a reduction of the convection coefficient. The other determining factor in the mechanism is the whether the surface being considered is the top surface or the bottom surface.

For most frequently encountered situations and for walls inclined from vertical with an angle θ between 0 and 60 degrees, the only change in the calculation of the convection coefficient is to replace g with g cos θ in (3). It should be noted that this is only appropriate for top and bottom surfaces of cooled and heated plates respectively.

3.3 Horizontal surfaces

For horizontal plates, the buoyancy forces are essentially normal to the plate. As for inclined surfaces the convection mechanism is influenced by whether the surface is cooled or heated and by whether the surface is facing upward or downward. For a cold surface facing downward and for a hot surface facing upward the convection is much more effective than in the opposite cases. In case of horizontal surfaces we define the characteristic length as the ratio between the area of the surface and the perimeter (8):

$$L = \frac{A}{P} \tag{8}$$

The formulas for the Nusselt number are:

Upper surface of heated plate or lower surface of cooled plate:

$$\begin{aligned} \overline{Nu} &= 0.54 \cdot Ra^{1/4} \qquad (10^4 \le Ra \le 10^7) \\ \overline{Nu} &= 0.15 \cdot Ra^{1/3} \qquad (10^7 \le Ra \le 10^{11}) \end{aligned} \tag{9}$$

Lower surface of heated plate or upper surface of cooled plate:

$$\overline{Nu} = 0.27 \cdot Ra^{1/4} \qquad (10^5 \le Ra \le 10^{10}) \tag{10}$$

3.4 Long horizontal cylinder

Equation (11) gives the formula valid for the isothermal long cylinder. The formula (11) is an average value valid for a large spectrum of Rayleigh numbers.

$$\overline{Nu} = \left\{ 0.6 + \frac{0.387 \cdot Ra^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}} \right\}^2 \qquad Ra \le 10^{12} \tag{11}$$

In the calculation regarding the Rayleigh number in (11) as characteristic length is considered the diameter D of the cylinder.

4. The case of forced convection

The physics of the heat transfer in the case of forced convection is similar to the natural (free) convection case. The first step in the process of calculating the convection coefficient is to determine the character (laminar or turbulent) of the flow since the convection coefficient depends strongly on which of these conditions exist.

Similarly to the situation encountered in the case of free convection, a velocity boundary layer is developed. The boundary layer is initially laminar but at some distance from the leading edge small disturbances appear, are amplified and after a transition region they develop into a completely turbulent flow region. Therefore we'll assume that at some distance (critical distance x_c) from the leading edge the turbulent flow occurs. This location is determined by the Reynolds number,

$$\mathrm{Re}_x = \frac{\rho \cdot u_\infty \cdot x}{\mu} = \frac{u_\infty \cdot x}{\nu} \qquad (12)$$

where u_∞ is the velocity of the free stream of fluid.

The physical interpretation of the Reynolds number is that it represents the ratio between inertia forces and viscous forces. In any flow there are some disturbances that can be amplified to lead to a turbulent flow. A small value of the Reynolds number means that the viscous forces are relatively important and the amplification of the disturbances is prevented. Also the thickness of the velocity boundary layer is related to the Reynolds number: at low values of the number we expect the thickness of the layer to increase.

The role of the Reynolds number in forced convection is similar to the role of the Grashof number in free convection. Note that the Grashof number is a measure of the ratio between buoyancy forces to viscous forces in the velocity boundary layer.

The critical value of the Reynolds number for which the flow over a flat plate transitions from a laminar to a turbulent flow varies between 10^5 and $3 \cdot 10^6$, depending on the roughness of the surface and the turbulence in the free stream. It is usual to consider for calculations a representative value of $5 \cdot 10^5$. With this value equation (12) can be used to determine the critical length where the transition begins. The location where the transition begins is more conventional than physical, in reality the transition between laminar flow and turbulent occurs in a buffer zone between the two. However the critical characteristic length (measured from the leading edge) can be used as an average value in calculations for the purpose of determining the appropriate convection coefficient. It is possible in practical applications to encounter situations where on the same surface the fluid flow transitions from a laminar flow to a turbulent flow. As a consequence it may be necessary to adjust the calculation of the convection coefficient accordingly.

There is another number which is important in forced convection. As in the case of free convection one has to consider the Prandtl number. The significance of the Prandtl number is that it represents the ratio between energy transport by diffusion in the velocity layer and the thermal layer. For gases the Prandtl number is close to 1, for liquid metals Pr << 1, for oils the opposite is true, Pr >> 1.

4.1 Laminar flow over flat plates

The convection coefficient is in general a function of distance (from the leading edge) mainly due to the fact that the Reynolds number (12) depends on x. Therefore we'll give both a local value and an average value for the Reynolds number.

Equation (13) is used for the laminar flow over isothermal plates for the local value of the Nusselt number. Equation (14) yields the average value of the Nusselt number. If the flow is laminar over the entire surface the "x" may be replaced by "L" and (14) can be used to predict average conditions over the entire surface.

$$Nu_x = \frac{h \cdot x}{k} = 0.332 \cdot \mathrm{Re}_x^{1/2} \cdot \mathrm{Pr}^{1/3} \qquad \mathrm{Pr} \geq 0.6 \qquad (13)$$

$$\overline{Nu} = \frac{\overline{h} \cdot x}{k} = 0.664\,\mathrm{Re}_x^{1/2} \cdot \mathrm{Pr}^{1/3} \qquad \mathrm{Pr} \geq 0.6 \qquad (14)$$

As for free convection all fluid properties are evaluated at film temperature using (4). Note that the average value given by (14) for a surface from the leading edge to a location "x" is twice the local value at that point "x".

For low Prandtl numbers (the case of liquid metals) equations (13) and (14) cannot be applied. In such cases (15) should be used:

$$Nu_x = 0.565 \cdot Pe_x^{1/2} \qquad \mathrm{Pr} \leq 0.05, \quad Pe_x \geq 100 \qquad (15)$$

where $Pe_x = \mathrm{Re}_x \cdot \mathrm{Pr}$ is the Peclet number

A single equation that can be applied for all Prandtl numbers (laminar flow over an isothermal plate) is presented in (16).

$$Nu_x = \frac{0.3387 \cdot \mathrm{Re}_x^{1/2} \cdot \mathrm{Pr}^{1/3}}{[1 + (0.0468/\mathrm{Pr})^{2/3}]^{1/4}} \qquad Pe_x \geq 100 \qquad (16)$$

While (16) can be used for local evaluations of the convection coefficient, the average Nusselt number can be calculated with $\overline{Nu}_x = 2 \cdot Nu_x$ as discussed above.

4.2 Turbulent flow over flat plates

The local Nusselt number for turbulent flow over an isothermal plate is given by:

$$Nu_x = 0.0296 \cdot \mathrm{Re}_x^{4/5} \cdot \mathrm{Pr}^{1/3} \qquad 0.6 \le \mathrm{Pr} \le 60 \tag{17}$$

As for the calculation of an average value for a plate with turbulent flow over its entire surface, it is acceptable to consider as average value the local value at the midpoint of the plate.

4.3 Mixed flow conditions over flat plates

In the case where there is a transition from laminar to turbulent flow the change in the respective convection coefficient is so large that the use of wrong formulas will likely cause large inaccuracy in the result. In this case an average value of the convection coefficient that takes into account the dual nature of the flow is a must. This task can be accomplished using an averaging equation (18):

$$\bar{h}_L = \frac{1}{L}\left(\int_0^{x_c} h_{lam} \cdot dx + \int_{x_c}^{L} h_{turb} \cdot dx\right) \tag{18}$$

where x_c is the critical length where the transition occurs between laminar and turbulent flow.

The result of the averaging operation is:

$$\overline{Nu} = [0.664 \cdot \mathrm{Re}_{x,c}^{1/2} + 0.037 \cdot (\mathrm{Re}_L^{4/5} - \mathrm{Re}_{x,c}^{4/5})] \cdot \mathrm{Pr}^{1/3} \quad 0.6 \le \mathrm{Pr} \le 60, \quad 5 \cdot 10^5 \le \mathrm{Re}_L \le 10^8 \tag{19}$$

with a typical value of $\mathrm{Re}_{x,c} = 5 \cdot 10^5$, (19) becomes:

$$\overline{Nu}_L (0.037 \cdot \mathrm{Re}_L^{4/5} - 871) \cdot \mathrm{Pr}^{1/3} \tag{20}$$

In situations for which $L \gg x_c$ ($\mathrm{Re}_L \gg \mathrm{Re}_{x,c}$), a reasonable approximation of (20) is (21):

$$\overline{Nu}_L = 0.037 \cdot \mathrm{Re}_L^{4/5} \cdot \mathrm{Pr}^{1/3} \tag{21}$$

4.4 Constant heat flux case (flat plate)

Results in the previous paragraphs dealing with forced convection are only valid for the case of isothermal plate. If the plate is subject to a uniform heat flux, the temperature distribution over the plate is not uniform. For laminar flow the appropriate formula for local Nusselt number is (22):

$$Nu_x = 0.453 \cdot \mathrm{Re}_x^{1/2} \cdot \mathrm{Pr}^{1/3} \qquad \mathrm{Pr} \geq 0.6 \tag{22}$$

while for turbulent flow it is (23)

$$Nu_x = 0.0308 \cdot \mathrm{Re}_x^{4/5} \cdot \mathrm{Pr}^{1/3} \qquad 0.6 \leq \mathrm{Pr} \leq 60 \tag{23}$$

4.5 Cylinder in cross flow

First of all let us specify that the Reynolds number is defined as:

$$\mathrm{Re}_D = \frac{u_\infty \cdot D}{\nu} \tag{24}$$

The equation to use for the entire range of Re as well as a wide range of Pr (all Re·Pr>0.2) is (25):

$$\overline{Nu}_D = 0.3 + \frac{0.62 \cdot \mathrm{Re}_D^{1/2} \cdot \mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}} \left[1 + \left(\frac{\mathrm{Re}_D}{282{,}000}\right)^{5/8} \right]^{4/5} \tag{25}$$

where all properties are evaluated at T_∞.

The technical literature mentions a great variety of empirical correlation formulas, which – as a rule – yield close values for the calculated quantities (such as convection coefficients). As an example for the cylinder in close flow another correlation widely used is given by (26):

$$\overline{Nu}_D = C \cdot \mathrm{Re}_D^m \cdot \mathrm{Pr}^n \left(\frac{\mathrm{Pr}}{\mathrm{Pr}_w} \right)^{1/4} \tag{26}$$

$0.7 < \mathrm{Pr} < 500$; $1 < \mathrm{Re}_D < 10^6$,

where all properties are evaluated at T_∞, except Pr_w, which is evaluated at wall temperature T_w.

Values of coefficients C and m are listed below. If Pr < 10, n = 0.37; if Pr > 10, n = 0.36.

ReD	C	m
1 – 40	0.75	0.4
40 – 1000	0.51	0.5
10^3 – 2 10^5	0.26	0.6
2 10^5 – 10^6	0.076	0.7

The relationship between the Nusselt number and the convection coefficient is similarly with the previous cases before,

$$\overline{Nu} = \frac{\overline{h} \cdot D}{k} \tag{27}$$

4.6 Steps for calculating the convection coefficient

Follow these simple steps to calculate the convection coefficient:

- determine the appropriate geometry (flow over plate, cylinder, etc);
- estimate the film temperature and evaluate all fluid properties at that temperature;
- calculate the Reynolds number, determine (for flat plate) if the flow is laminar or turbulent;
- decide on the formula to use (local or average coefficient).

5. Combined natural (free) and forced convection

To obtain an indication of the relationship between free convection and forced convection one can use the ratio between Grashof number and the square of Reynolds number.
In other words the above mentioned ratio gives a qualitative indication of the influence of the buoyancy forces on forced convection. When the Grashof number is of the same order of magnitude or larger than the square of the Reynolds number, free convection effects cannot be ignored, compared with the effects of forced convection. Similarly, when the square of Reynolds number is of the same order of magnitude as the Grashof number forced convection has to be taken into account together with natural convection. There are three main cases corresponding to different combinations of free convection effects and forced convection effects. If the buoyancy-induced motion and forced motion have the same direction we have a case of *assisting flow*. If the two flows are in opposite directions we have a *transversal flow*.

$$Nu^n_{composite} = Nu^n_{Forced} \pm Nu^n_{Natural} \tag{28}$$

In (28) the + sign applies for assisting and transverse flow cases, the – sign applies for the case of opposing flow. The forced and natural components are determined according to the respective procedures indicated in the previous chapters from the existing formulas. For “n” the most used value is n = 3, however for the case of transverse flow over plat plates and cylinders n=3.5 may provide a better correlation of data.

Note that (28) should be seen as a first approximation for the mixed convection which is a rather complicated phenomenon. Finally we note that although buoyancy effects can significantly enhance heat transfer for laminar forced convection, enhancement is typically negligible if the forced flow is turbulent.

6. Conclusion

There is one important aspect that needs to be highlighted. In all calculations of the fluid properties the film temperature has to be used. The film temperature depends however of the temperature of the solid (wall) that we want to calculate as part of the thermal problem. This means that prior to the calculation of the convection coefficient the temperature of the surface exhibiting convection has to be estimated. Then the film temperature and the convection coefficient can be calculated. Once the thermal problem is solved (using the convection coefficient estimated as above) another iteration may be initiated by re-calculating the film temperature, a new convection coefficient and doing another thermal simulation.

Another remark is that the process of calculation of convection coefficients based on empirical formulas can cause deviations of 15-25 % or more. One should keep in mind that the convection process is a very complex one as described in its fundamental aspects in the present document. For accurate predictions the integration of boundary layer equations has to be included in the computational process.

7. Examples

We present a number of examples of how to use some of the formulas presented in the previous chapters. The examples include cases of free (natural) convection and cases of forced convection.

7.1 Convection between a warm wall and ambient

We consider the wall with height of L = 0.71 m, width W = 1.02 m at uniform temperature $T_w = 232 C^{\circ}$. The air in the room is quiescent at 23 C°.

Fig. 3 Free convection example

Air properties are evaluated at film temperature, T_f = 400 K. They are:

k = 33.8 10^{-3} W/mK,

$\nu = 26.4 \cdot 10^{-6} m^2 / s$

$\alpha = 38.3 \cdot 10^{-6} m^2 / s$

$\Pr = \frac{\nu}{\alpha} = 0.69$

$\beta = \frac{1}{T_f} = 0.0025 K^{-1}$

With the above properties we can now calculate the convection coefficient. From (2) and (3) one obtains from the Rayleigh number:

Ra = 1.813 10^9, and from paragraph 3 it follows that transition to turbulent flow occurs on the wall. As a consequence equation (7a) will be used to yield:

$$\overline{Nu} = \left\{ 0.825 + \frac{0.387 \cdot Ra^{1/6}}{[1 + (0.492 / \Pr)^{9/16}]^{8/27}} \right\}^2 = 147$$

From (6) one obtains:

$$\overline{h} = \frac{\overline{Nu} \cdot k}{L} = \frac{147 \cdot 33.8 \cdot 10^{-3} \; W / m \cdot K}{0.71 m} = 7 \; W / m^2 \cdot K$$

Using Newton’s law of cooling (1), the power (heat rate) transferred from the wall to the room air is

$$q = 7.0 \cdot (1.02 \cdot 0.71)(232 - 23) = 1{,}060 W$$

7.2 Free convection between an air duct and ambient

Let us consider the situation presented in Fig. 4 where the airflow through a duct 0.75 m wide and 0.3 m high maintains the outer duct surface at 45 C. The ambient air is assumed quiescent at room temperature of 15 C.

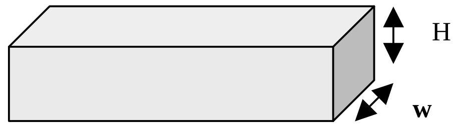

Fig. 4 Geometry of the duct

At T_f = 303 K the air properties are:

$$\nu = 16.2 \cdot 10^{-6} m^2 / s$$

$$\alpha = 22.9 \cdot 10^{-6} m^2 / s$$

$$\Pr = \frac{\nu}{\alpha} = 0.71$$

$$\beta = \frac{1}{T_f} = 0.0033 K^{-1}$$

$$k = 0.0265 W / m$$

This is a case where the convection is considered separately from the sides as well as the other two horizontal surfaces (top and bottom). As previously discussed different formulas are used as appropriate.

With the above one obtains for the Rayleigh number Ra = 2.62 L^3.

For the two sides L = H = 0.3 m, which yields Ra = 7.07 10^7, corresponding to a laminar flow in the boundary layer. Therefore (7b) is used to obtain:

$$h_{side} = \frac{k}{H} \cdot \overline{Nu} = \frac{0.0265}{0.3} \left\{ 0.68 + \frac{0.67 \cdot (7.07 \cdot 10^7)^{1/4}}{[1 + (0.492/0.71)^{9/16}]^{4/9}} \right\} = 4.23 W / m^2 K$$

For the top and bottom surfaces with (8) one gets L ~ w/2 = 0.375 m and therefore Ra = 1.38 10^8.

Using the second equation (9) and equation (10) one obtains:

$$\overline{h}_{top} = \frac{k}{w/2} \cdot 0.15 \cdot Ra^{1/3} = 5.47 W / m^2 K$$

$$\overline{h}_{bottom} = \frac{k}{w/2} 0.27 \cdot Ra^{1/4} = 2.07 W / m^2 K$$

7.3 Forced convection of air over an isothermal flat plate

Assume air at a pressure of 6 kN/m^2 and 300 C flows with a velocity of 10 m/s over a flat plate 0.5 m long and 27 C surface temperature. It is clear that the convection occurs to transfer heat *to* the plate and not *from* the plate.

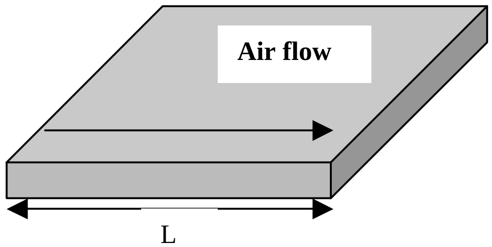

Fig. 5 Forced air convection over a flat plate

Gas properties are available from tables usually at p = 1 atm (at varying temperatures). The majority of properties such as k, Pr, μ are with a very good approximation independent of the pressure. However others such as ν depend on pressure. The kinematic viscosity varies with pressure because the density varies with pressure. According to the ideal gas law:

$$\rho = \frac{p}{R \cdot T}$$

Since $\nu = \dfrac{\mu}{\rho}$

It follows that at the same temperature but at different pressure one has:

$$\frac{\nu_1}{\nu_2} = \frac{p_2}{p_1}$$

and therefore the kinematic viscosity of air at 437 K and 6,000 N/m^2 is:

$$\nu = 30.84 \cdot 10^{-6} \cdot \frac{1.0133 \cdot 10^5}{6000} = 5.21 \cdot 10^{-4} \ m^2 / s$$

With (12) the Reynolds number is Re = 9,597 which corresponds to a case of laminar flow over the entire surface of the plate. From (14) the average Nusselt number is:

$$\overline{Nu} = 0.664 \cdot \mathrm{Re}^{1/2}\, \mathrm{Pr}^{1/3} = 57.4$$

$$\bar{h} = \frac{\overline{Nu} \cdot k}{L} = 4.18\, W / m^2 \cdot K$$

7.4 Cross flow forced convection of air over a cylinder

Consider a cylinder 12.7 mm in diameter and 94 mm long placed in a wind tunnel. The air flow in the wind tunnel occurs at 10 m/s and the air temperature is 26.2 C. There is a power dissipation inside the cylinder due to an electric resistance. As a result, the average temperature the cylinder is 128.4 C.

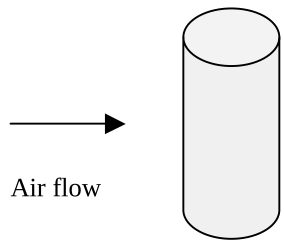

Fig. 6 Cross flow of air over heated cylinder

For this application one has to get the air properties at three temperatures: the temperature of free stream of air, film temperature and wall temperature.

At $T_\infty = 26.2\, C (\approx 300\, K)$:

$$\nu = 15.89 \cdot 10^{-6}\, m^2 / s$$
$$k = 26.3 \cdot 10^{-3} W / m \cdot K$$
$$\Pr = 0.707$$

At T_f = 350 K:

$$\nu = 20.92 \cdot 10^{-6}\, m^2 / s$$
$$k = 30 \cdot 10^{-3} W / m \cdot K$$
$$\Pr = 0.7$$

At T_w = 401 K

$\Pr_w = 0.69$

The Reynolds number:

$$\mathrm{Re} = \frac{u_\infty \cdot D}{\nu} = \frac{10 \cdot 0.0127}{20.92 \cdot 10^{-6}} = 6{,}071$$

The Nusselt number is calculated with (25) with all properties evaluated at T_f:

$$\overline{Nu}_D = 0.3 + \frac{0.62 \cdot \mathrm{Re}_D^{1/2} \cdot \mathrm{Pr}^{1/3}}{[1+(0.4/\mathrm{Pr})^{2/3}]^{1/4}} \left[1 + (\frac{\mathrm{Re}_D}{282,000})^{5/8} \right]^{4/5} = 40.6$$

$$\overline{h} = \overline{Nu} \cdot \frac{k}{D} = 96\, W / m^2 K$$

When using (26) the values for the constants are C = 0.26, m = 0.6. Since Pr < 10, n = 0.37. With (26) one calculates:

$$\overline{Nu}_D = 0.26 \cdot (7992)^{0.6} \cdot (0.707)^{0.37} \left(\frac{0.707}{0.69} \right)^{0.25} = 50.5$$

$$\overline{h} = \overline{Nu}_D \cdot \frac{k}{D} = 105\, W / m^2 \cdot K$$

One can see the different value from the one obtained with (25). This is only shown here to make clearer the point that some variation should be expected when calculating convection coefficients with different formulas. Also note that in the result above the wall temperature is involved due to T_w.