```
> restart:
> with(plots):with(plottools):with(linalg): with(geom3d):
Warning, the name changecoords has been redefined

Warning, the name arrow has been redefined

Warning, the protected names norm and trace have been redefined and unprotected

Warning, these names have been redefined: circle, dodecahedron, hexahedron, homothety,
icosahedron, inverse, line, octahedron, point, polar, reflect, rotate, sphere,
stellate, tetrahedron, transform, translate
```

Vektor aus dem Winkel alpha

```
> point(p,0,0,0):
> ov:=Vector([0,0,0]);
```

$$ov := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

```
> vect_a:=Vector([1,0,0]);
> vect_b:=Vector([cos(alpha),0,sin(alpha)]);
```

$$vect\_a := \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$vect\_b := \begin{bmatrix} \cos(\alpha) \\ 0 \\ \sin(\alpha) \end{bmatrix}$$

```
>
>
```

E1: Ebene aus Normalenvektor: E1= <u>n</u>*(<u>x</u>-<u>p</u>) -> <u>p</u>=[0,0,0] -> E1=<u>n</u>*<u>x</u>

```
> E1:=matrix([ [0,0,(sin(alpha))] , [0,1,0] , [0,0,cos(alpha)] ]);
```

$$E1 := \begin{bmatrix} 0 & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ 0 & 0 & \cos(\alpha) \end{bmatrix}$$

>

E2: Ebene, um Z-Achse drehabar

```
> E2:= matrix([[0,0,sin(delta)],[0,0,cos(delta)],[0,1,0]]);
```

$$E2 := \begin{bmatrix} 0 & 0 & \sin(\delta) \\ 0 & 0 & \cos(\delta) \\ 0 & 1 & 0 \end{bmatrix}$$

E3: Ebene, gebildet aus der X und Y-Achse

```
> E3:= matrix([[0,1,0],[0,0,1],[0,0,0]]);
```

$$E3 := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

>

>

Schnittgerade (vect_c) von E1 und E3

```
> vect_c:=Vector([-sin(delta),cos(delta),0]);
```

$$vect\_c := \begin{bmatrix} -\sin(\delta) \\ \cos(\delta) \\ 0 \end{bmatrix}$$

>

Schnittgerade (vect_d) von E2 und E3

>

```
> zw_1:=
matrix([[0,sin(alpha),0,-sin(delta),0],[1,0,0,-cos(delta),0],[0,cos(al
pha),-1,0,0]]);
>
```

$$zw\_1 := \begin{bmatrix} 0 & \sin(\alpha) & 0 & -\sin(\delta) & 0 \\ 1 & 0 & 0 & -\cos(\delta) & 0 \\ 0 & \cos(\alpha) & -1 & 0 & 0 \end{bmatrix}$$

```
> zw_2:=gaussjord(zw_1);
```

```
Warning, unable to find a provably non-zero pivot
```

$$zw\_2 := \begin{bmatrix} 1 & 0 & 0 & -\cos(\delta) & 0 \\ 0 & 1 & 0 & -\frac{\sin(\delta)}{\sin(\alpha)} & 0 \\ 0 & 0 & 1 & -\frac{\cos(\alpha)\sin(\delta)}{\sin(\alpha)} & 0 \end{bmatrix}$$

```
> r3:=(solve(  zw_2[3,3]*r3  +  zw_2[3,4] = 0,  r3 ));
```

$$r3 := \frac{\cos(\alpha)\sin(\delta)}{\sin(\alpha)}$$

```
> vect_d:=((Vector([0,0,r3])+Vector([-sin(delta),cos(delta),0])));
```

$$vect\_d := \begin{bmatrix} -\sin(\delta) \\ \cos(\delta) \\ \frac{\cos(\alpha)\sin(\delta)}{\sin(\alpha)} \end{bmatrix}$$

```
>
```

Winkel zwischen zwei Vektoren

```
> Erg1:=(cos(beta)=(  vect_d[1]*vect_c[1] +  vect_d[2]*vect_c[2]  +
  vect_d[3]*vect_c[3]          )  /  (norm(vect_c,2)*norm(vect_d,2)));                    simplif
  y(Erg1);
```

$$Erg1 := \cos(\beta) = \frac{\sin(\delta)^2 + \cos(\delta)^2}{\sqrt{|\sin(\delta)|^2 + |\cos(\delta)|^2}\,\sqrt{|\sin(\delta)|^2 + |\cos(\delta)|^2 + \left|\frac{\cos(\alpha)\sin(\delta)}{\sin(\alpha)}\right|^2}}$$

$$\cos(\beta) = \frac{1}{\sqrt{|\sin(\delta)|^2 + |\cos(\delta)|^2}\,\sqrt{|\sin(\delta)|^2 + |\cos(\delta)|^2 + \left|\frac{\cos(\alpha)\sin(\delta)}{\sin(\alpha)}\right|^2}}$$

```
>
```

Entfernen der Betragszeichen.

```
> cos(beta) =
  1/(((sin(delta))^2+(cos(delta))^2)^(1/2)*((sin(delta))^2+(cos(delta))^
  2+(cos(alpha)*sin(delta)/sin(alpha))^2)^(1/2));
```

$$\cos(\beta) = \frac{1}{\sqrt{\sin(\delta)^2 + \cos(\delta)^2}\,\sqrt{\sin(\delta)^2 + \cos(\delta)^2 + \frac{\cos(\alpha)^2 \sin(\delta)^2}{\sin(\alpha)^2}}}$$

\>

```
> (solve({cos(beta) = 
1/(sin(delta)^2+cos(delta)^2)^(1/2)/(sin(delta)^2+cos(delta)^2+cos(alpha)^2*sin(delta)^2/sin(alpha)^2)^(1/2)},{delta}));
```

$$\left\{\delta = \arctan\left(\frac{\sqrt{1-\cos(\beta)^2}\tan(\alpha)}{\cos(\beta)}, \frac{\sqrt{\tan(\alpha)^2\cos(\beta)^2+\cos(\beta)^2-\tan(\alpha)^2}}{\cos(\beta)}\right)\right\},$$
$$\left\{\delta = \arctan\left(-\frac{\sqrt{1-\cos(\beta)^2}\tan(\alpha)}{\cos(\beta)}, \frac{\sqrt{\tan(\alpha)^2\cos(\beta)^2+\cos(\beta)^2-\tan(\alpha)^2}}{\cos(\beta)}\right)\right\},$$
$$\left\{\delta = \arctan\left(\frac{\sqrt{1-\cos(\beta)^2}\tan(\alpha)}{\cos(\beta)}, -\frac{\sqrt{\tan(\alpha)^2\cos(\beta)^2+\cos(\beta)^2-\tan(\alpha)^2}}{\cos(\beta)}\right)\right\},$$
$$\left\{\delta = \arctan\left(-\frac{\sqrt{1-\cos(\beta)^2}\tan(\alpha)}{\cos(\beta)}, -\frac{\sqrt{\tan(\alpha)^2\cos(\beta)^2+\cos(\beta)^2-\tan(\alpha)^2}}{\cos(\beta)}\right)\right\}$$

HIER TESTWINKEL EINGEBEN

\>

```
> alpha:=30*Pi/180;
```

$$\alpha := \frac{1}{6}\pi$$

```
> beta:=30*Pi/180;
```

$$\beta := \frac{1}{6}\pi$$

\>

Delta ist die Summe aus Alpha' und Beta'

```
> (solve({cos(beta) = 
1/(sin(delta)^2+cos(delta)^2)^(1/2)/(sin(delta)^2+cos(delta)^2+cos(alpha)^2*sin(delta)^2/sin(alpha)^2)^(1/2)},{delta}));
```

$$\left\{\delta = -\arctan\left(\frac{1}{4}\sqrt{2}\right)\right\}, \left\{\delta = \arctan\left(\frac{1}{4}\sqrt{2}\right) - \pi\right\}, \left\{\delta = \arctan\left(\frac{1}{4}\sqrt{2}\right)\right\},$$

$$\left\{ \delta = -\arctan\left(\frac{1}{4}\sqrt{2}\right) + \pi \right\}$$

>

>

Crossprodukt **b*d**

>

Aus dem Kreuzprodukt von Vektor b und d und mit dem nun bekannten Delta lässt sich der Vektor e , und daraus der Winkel Gamma bestimmen.

```
> delta:=arctan((1-cos(beta)^2)^(1/2)*tan(alpha)/cos(beta),
(tan(alpha)^2*cos(beta)^2+cos(beta)^2-tan(alpha)^2)^(1/2)/cos(bet
a));   evalf[5](delta*180/Pi);
```

$$\delta := \arctan\left(\frac{1}{8}\sqrt{4}\sqrt{2}\right)$$

$$19.471$$

>

```
> vect_b;vect_d;
```

$$\begin{bmatrix} \cos(\alpha) \\ 0 \\ \sin(\alpha) \end{bmatrix}$$

$$\begin{bmatrix} -\sin(\delta) \\ \cos(\delta) \\ \frac{\cos(\alpha)\sin(\delta)}{\sin(\alpha)} \end{bmatrix}$$

>

```
> vect_e:=Vector    ( crossprod(vect_b,vect_d)):          vect_e:
```

>

>

Winkel zwischen zwei Vektoren

```
> Erg2:=(cos(phi)=(  vect_e[1]*0 +  vect_e[2]*0  + vect_e[3]*1  )
/   (norm(vect_e,2)*norm([0,0,1],2)));
```

$$Erg2 := \cos(\phi) = \frac{1}{2}\sqrt{2}$$

```
> 
> Erg3:= evalf[5]((solve(Erg2),phi));
```

$$Erg3 := 0.78540$$

```
> 
> 
```

Ergebnis mit z.B. Alpha = 30° und Beta = 30°

```
> Alpha_=evalf[5](alpha*180/Pi); Beta_=evalf[5](beta*180/Pi);
Alpha_und_Beta_Strich=evalf[5](delta*180/Pi); phi_=evalf[5](Erg3*180/P
i);
```

$$Alpha\_ = 30.$$

$$Beta\_ = 30.$$

$$Alpha\_und\_Beta\_Strich = 19.471$$

$$phi\_ = 44.999$$