

Taylor-Entwicklung für den natürlichen Logarithmus

$$\ln(1-z) \xrightarrow{\text{series}} -z - \frac{z^2}{2} - \frac{z^3}{3} - \frac{z^4}{4} - \frac{z^5}{5} - \frac{z^6}{6}$$

$$\ln(1+z) \xrightarrow{\text{series}} z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \frac{z^5}{5} - \frac{z^6}{6}$$

$$\ln\left(\frac{1+z}{1-z}\right) \xrightarrow{\text{series}} 2 \cdot z + \frac{2 \cdot z^3}{3} + \frac{2 \cdot z^5}{5}$$

$$\ln\left(\frac{1+z}{1-z}\right) = (\ln(1+z) - \ln(1-z)) = 2 \cdot z + \frac{2 \cdot z^3}{3} + \frac{2 \cdot z^5}{5}$$

$$x = \frac{1+z}{1-z} \quad z = \frac{x-1}{x+1}$$

$$\ln\left(\frac{1+z}{1-z}\right) = \ln(x) = 2 \cdot z + \frac{2 \cdot z^3}{3} + \frac{2 \cdot z^5}{5} = 2 \cdot \left(\frac{x-1}{x+1}\right) + \frac{2 \cdot \left(\frac{x-1}{x+1}\right)^3}{3} + \frac{2 \cdot \left(\frac{x-1}{x+1}\right)^5}{5}$$

$$\ln(x) = 2 \cdot \left(\left(\frac{x-1}{x+1}\right) + \frac{1}{3} \cdot \left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5} \cdot \left(\frac{x-1}{x+1}\right)^5 \right) = 2 \cdot \sum_{k=1}^n \frac{1}{2 \cdot k - 1} \cdot \left(\frac{x-1}{x+1}\right)^{2 \cdot k - 1}$$