Network Analyzer

Error Models

and

Calibration Methods

by

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This paper is an overview of error models and calibration methods for vector network analyzers.

Presentation Outline

- Network Analyzer Block Diagram and Error Model
- System Error Model for Error-Correction
- One-Port Error Model and Calibration
- Two-Port Error Models and Calibration
  - 12-Term Method
  - 16-Term Method
  - 8-Term Method
- Measuring S-parameters
- Accuracy of Error-Correction

A system error model will be derived from a generic network analyzer block diagram. This error model will then be simplified to the standard one-port and 12-term two-port models used the past 30 years.

Newer 16-term and 8-term models will then be introduced and the modern calibration approaches described.

Measuring s-parameters without a Z_0 termination will be covered. Also the accuracy of error-corrected measurements will be outlined.

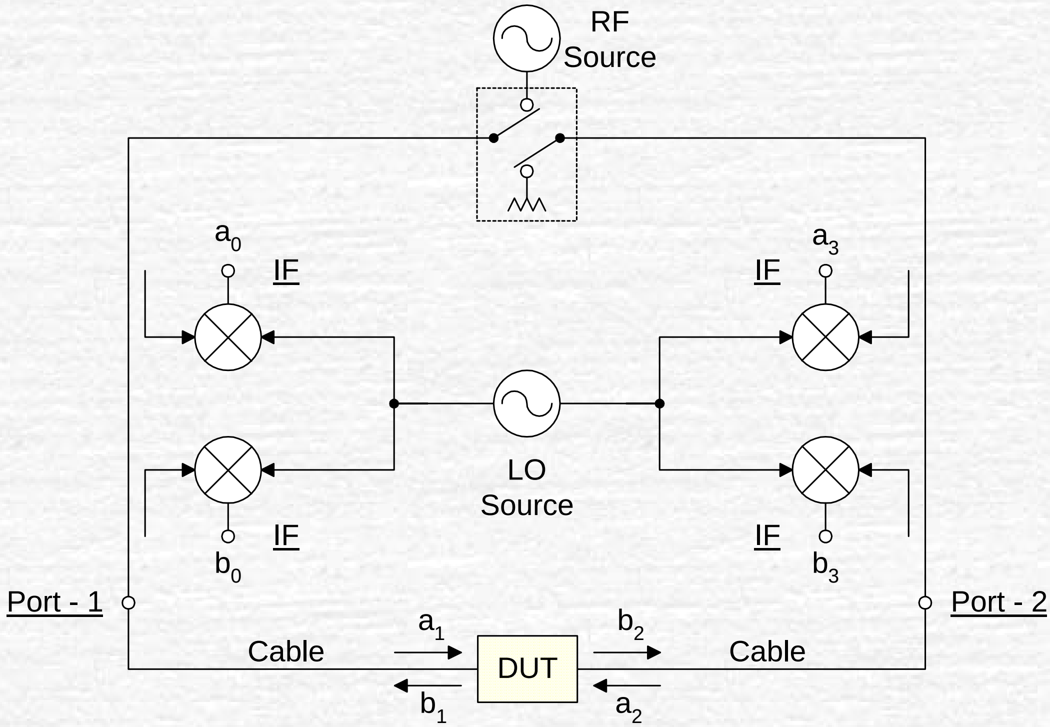
Network Analyzer Block Diagram

and

Error Model

First the block diagram for a network analyzer is described and the hardware flow graph is defined.

Network Analyzer Block Diagram



This is a generic block diagram of a 4 channel network analyzer. The source can be switched to excite port-1 or port-2 of the device under test (DUT). The switch also provides a Z_0 termination for the output port in each direction. Directional couplers are used to separate the incident, reflected and transmitted waves in both the forward and reverse direction. Mixers are used to down convert the RF signals to a fixed low frequency IF. The LO source is tuned to the frequency of the RF + IF.

The s-parameters of the DUT can be defined as follows:

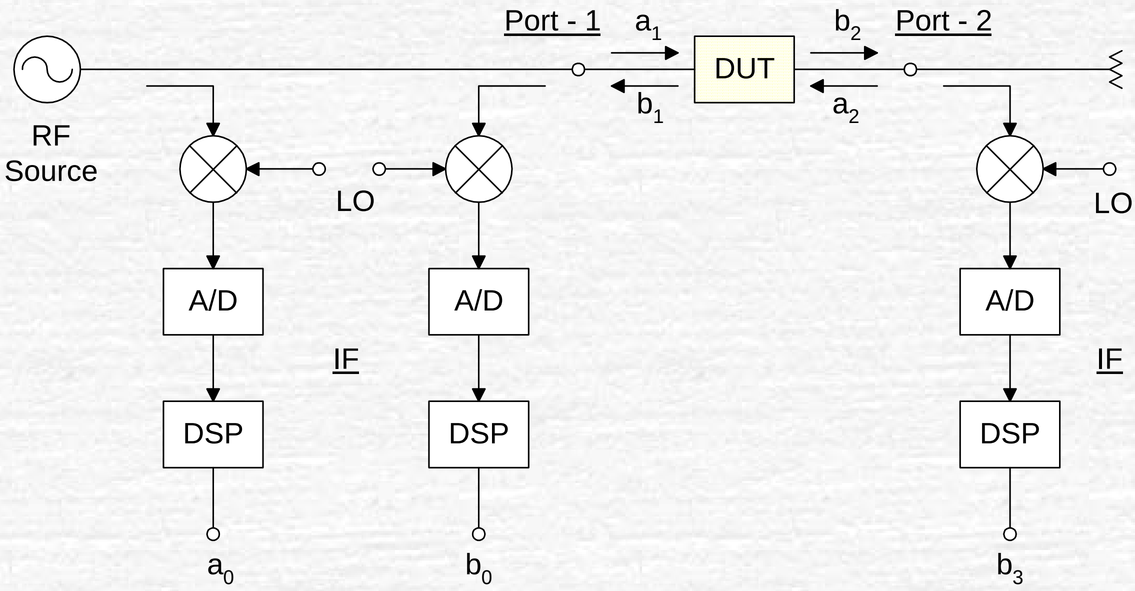
$S_{11} = b_1/a_1$, switch in forward direction

$S_{21} = b_2/a_1$, switch in forward direction

$S_{12} = b_1/a_2$, switch in reverse direction

$S_{22} = b_2/a_2$, switch in reverse direction

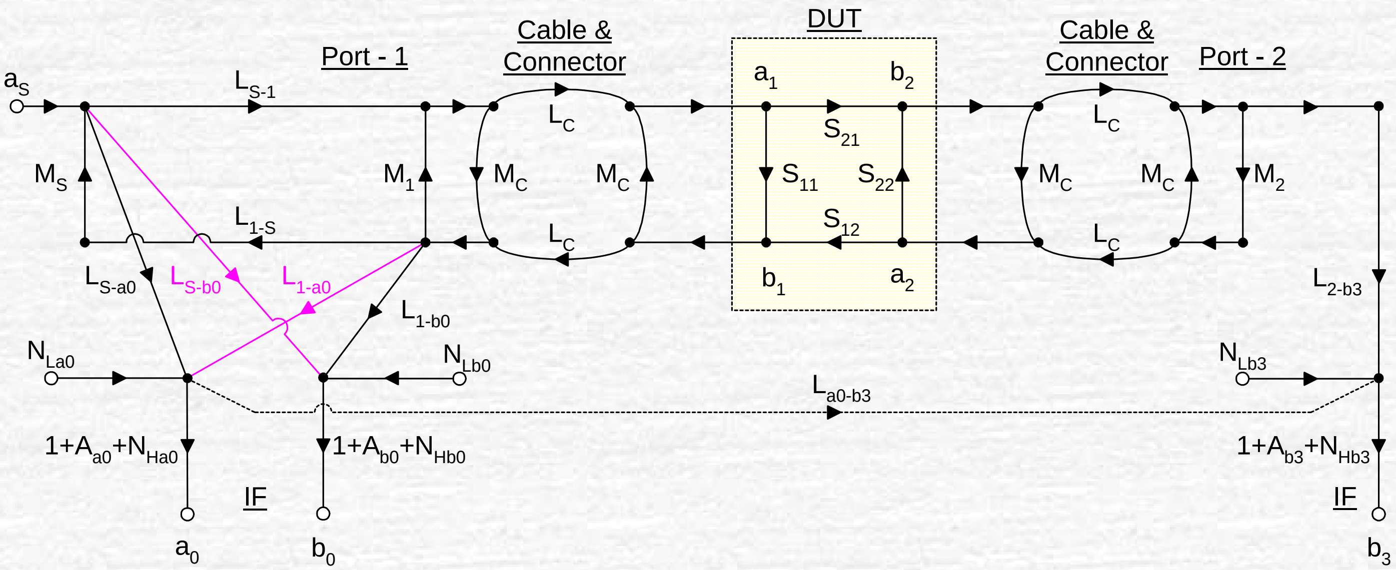
Block Diagram – Forward Direction



This block diagram shows the measurement system switched to the forward direction. Each of the IF signals are detected and digitized and the real and imaginary terms are measured. From this data the magnitude and phase can be calculated.

In most modern network analyzers the A/D digitizes directly at the IF and the detection is done in the digital domain. The resultant digitized versions of the DUT waves (a_0, b_0, and b_3) are a scaled version of the actual waves at the DUT (a_1, b_1, and b_2).

Error Model – Forward Direction



Branches With No Label = 1

From the block diagram a flowgraph can be developed showing all the possible signal paths. These paths not only include the main desired signals but the loss, match errors, and leakage errors, of the network analyzer along with the cables, connectors, or probes that connect to to DUT.

Also included in this model are the IF, A/D and detector non linearities and the system noise.

Error Model Definitions

a_1 = Incident Signal at Port-1
b_1 = Reflected Signal at Port-1
a_2 = Incident Signal at Port-2
b_2 = Transmitted signal at Port-2

a_S = Source Port
a_0 = Measured Incident Port
b_0 = Measured Reflected Port
b_3 = Measured Transmitted Port

L_{S-1} = Loss from Source to Port-1
L_{1-S} = Loss from Port-1 to Source
L_{S-a0} = Loss from Source to a_0
L_{S-b0} = Loss from Source to b_0 (Directivity)
L_{1-a0} = Loss from Port-1 to a_0 (Directivity)
L_{1-b0} = Loss from Port-1 to b_0
L_{2-b3} = Loss from Port-2 to b_3
L_{a0-b3} = Loss from a_0 to b_3 (Leakage)
L_C = Loss of Cables

S_{11} = Refl Coef of DUT at Port-1
S_{21} = Forward Trans Coef of DUT
S_{12} = Reverse Trans Coef of DUT
S_{22} = Refl Coef of DUT at Port-2

M_1 = Match at Port-1
M_2 = Match at Port-2
M_S = Match of Source
M_C = Match of Cables

N_{La0} = Low Level Noise at a_0
N_{Lb0} = Low Level Noise at b_0
N_{Lb3} = Low Level Noise at b_3
N_{Ha0} = High Level Noise at a_0
N_{Hb0} = High Level Noise at b_0
N_{Hb3} = High Level Noise at b_3

A_{a0} = Dynamic Accuracy at a_0 (Linearity)
A_{b0} = Dynamic Accuracy at b_0 (Linearity)
A_{b3} = Dynamic Accuracy at b_3 (Linearity)

The above table gives the description of each of the branches and the key nodes for the flow graph. This provides a very complete model for the network analyzer. However it is possible to reduce the flow graph without any loss in accuracy. This reduced flow graph is much easier to analyze and will be discussed next.

System Error Model

for

Error-Correction

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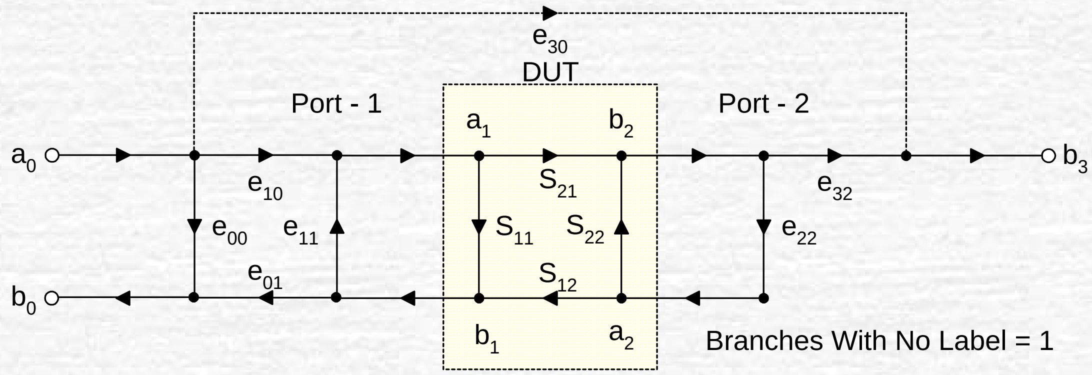
The simplified system error model is described. This system model will be used to develop the error correction procedure.

System Model – Forward Direction



Directivity	$e_{00} \cong \frac{L_{S-b0}}{L_{S-a0}}$	Leakage	$e_{30} \cong L_{a0-b3}$
Reflection Tracking	$e_{10}e_{01} \cong \frac{L_{S-1}L_{1-b0}}{L_{S-a0}} - \frac{L_{S-1}L_{1-a0}L_{S-b0}}{(L_{S-a0})^2}$	Transmission Tracking	$e_{10}e_{32} \cong \frac{L_{S1}L_{2-b3}}{L_{S-a0}}$
Port-1 Match	$e_{11} \cong M_2 - \frac{L_{S-1}L_{1-a0}}{L_{S-a0}}$	Port-2 Match	$e_{22} \cong M_2$

There are also Errors caused by the Converter, IF, Cables and Connectors

There are also Six Terms in the Reverse Direction

The resultant system error model is the forward portion of the well known 12-term error mode. Each of the branches have an accurate relationship to the original hardware oriented flow graph presented earlier. The 6 forward terms described above show a simplified set of equations relating the two flow graphs.

The directivity error is caused primarily by the coupler leakage or ‘coupler directivity.’ This error is also increased by cable and connector match errors between the measurement coupler and the DUT. The reflection and transmission tracking is caused by reflectometer and mixer tracking as well as cable length imbalance between the measured ports. The match error is the mathematical ratioed port match error that is not necessarily the ‘raw’ port match. The leakage error is through the LO path of the mixers. It is not the leakage of the switch and this model assumes the switch leakage is negligible.

Improvements with Correction

ERRORS REMOVED	ERRORS REMAINING
Port Match	Noise and Residuals
Directivity	Receiver Linearity
Tracking	Drift after Error-Correction
Main Leakage Paths	Stability after Error-Correction
	Repeatability of Connectors, etc
	Lower Lever Leakage Paths
	Errors of Calibration Standards

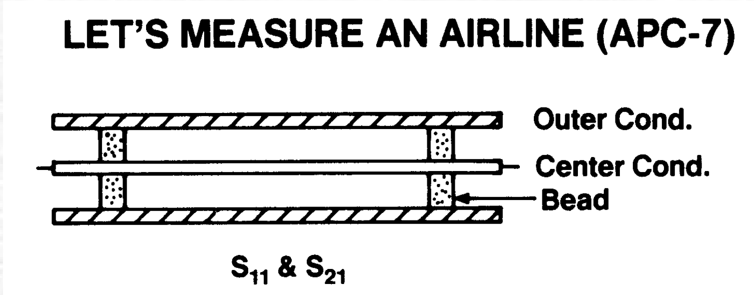
A linear calibration procedure is applied to remove as many of the errors as possible. The loss, directivity, match, and main leakage errors can be greatly reduced depending on the accuracy of the calibration standards used.

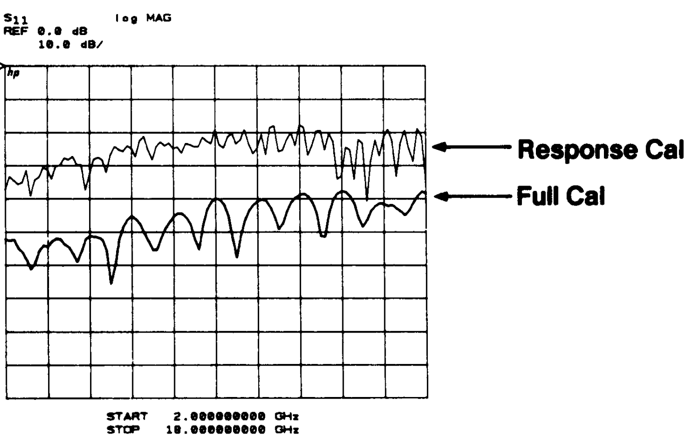
However, the noise and linearity errors can not be reduced using a simple linear calibration procedure. If fact the noise and linearity errors increase a small amount. Residuals also remain that are caused by A/D quantization errors, clock leakage, etc.

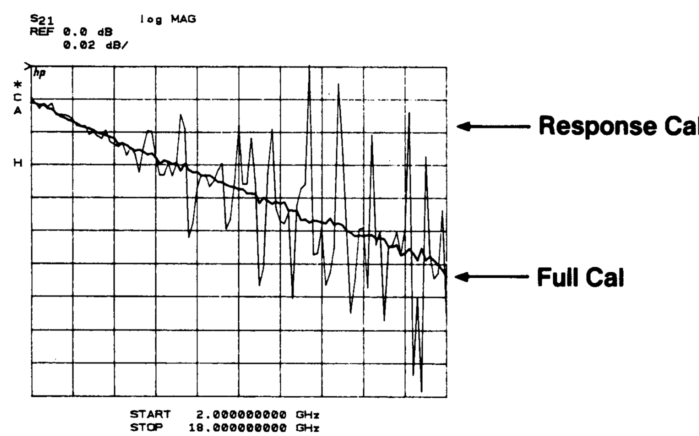
Once the network analyzer is calibrated the drift, stability, and repeatability errors will degrade the system performance. This usually means that the system will need to be recalibrated at some interval depending on the system usage, environment and required accuracy.

There are some lower level leakage paths in the RF hardware that are not modeled in many of the error correction schemes.

Improvements with Correction







To see the improvements offered by error correction lets compare the measurement results before and after error correction when measuring a beaded airline. The two test cases will be with a response only calibration, which will not remove the port match and directivity errors, and a full two port calibration which will remove all the errors.

There is a twenty dB improvement in the reflection measurements when the directivity errors are reduced.

The transmission tracking errors are greatly improved with error correction. The error was mainly caused by port match. It should be pointed out that the uncorrected response errors are only a tenth dB which means the uncorrected return loss at both test ports must be at least 20 dB. After correction they have been reduced 20 db more.

One Port Error Model

and

Calibration

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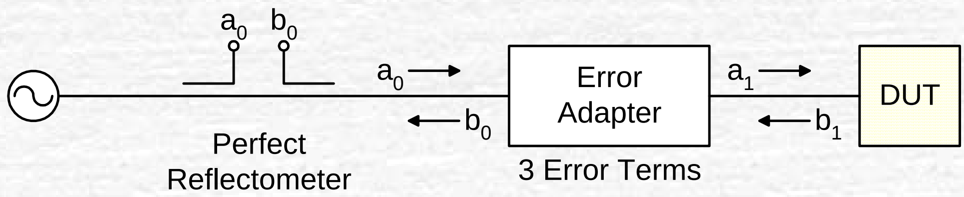
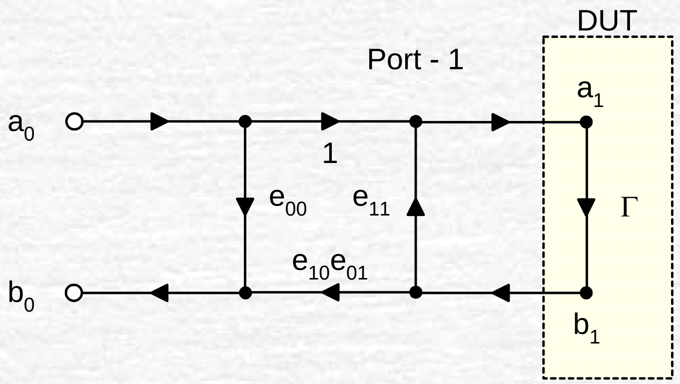
The one-port model will be first developed. This will then be used to further develop the two-port model.

One Port: 3-Term Error Model





e_{00} = Directivity

e_{11} = Port Match

$(e_{10}e_{01})$ = Tracking

The most simple case is the one port 3-term error model. From this model the same approach will be used for the two port cases.

In the one-port calibration procedure the model simplifies to just the terms describing the directivity, port match , and tracking errors at port-1.

The errors can be lumped into a fictitious error adapter that modifies the actual DUT reflection coefficient which is then measured by a ‘perfect’ reflectometer.

One Port: 3-Term Error Model

Measured | **Actual**

$$\Gamma_M = \frac{b_0}{a_0} = \frac{e_{00} - \Delta_e \Gamma}{1 - e_{11}\Gamma}$$

$$\Gamma = \frac{\Gamma_M - e_{00}}{\Gamma_M e_{11} - \Delta_e}$$

$$\Delta_e = e_{00}e_{11} - (e_{10}e_{01})$$

For ratio measurements there are 3 error terms
The equation can be written in the linear form

$$e_{00} + \Gamma\Gamma_M e_{11} - \Gamma\Delta_e = \Gamma_M$$

With 3 different known Γ, measure the resultant 3 Γ_M
This yields 3 equations to solve for e_{00}, e_{11}, and Δ_e

$$e_{00} + \Gamma_1\Gamma_{M1}e_{11} - \Gamma_1\Delta_e = \Gamma_{M1}$$

$$e_{00} + \Gamma_2\Gamma_{M2}e_{11} - \Gamma_2\Delta_e = \Gamma_{M2}$$

$$e_{00} + \Gamma_3\Gamma_{M3}e_{11} - \Gamma_3\Delta_e = \Gamma_{M3}$$

Any 3 independent measurements can be used

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Solving the one-port flow graph yields a bilinear relationship between the actual and measured reflection coefficient. The actual reflection coefficient is ‘mapped’ or modified by the three error terms to the measured result. This equation can be inverted to solve for the actual reflection coefficient knowing the measured result and the three error terms.

The three error terms can be determined by measuring three known standards (such as an open, short and load) that yield three simultaneous equations. These three equations can then be solved for the three error terms.

Another popular calibration standard is called e-cal. With this technique the reflection coefficient is selected electronically from a set of pre measured states. These were measured using a network analyzer calibrated with primary standards. This is strictly a transfer standard but as long as the e-cal standard is stable it is very accurate. This type of standard allows fast calibrations, ease of use and less operator error.

Two-Port Error Models

and

Calibration

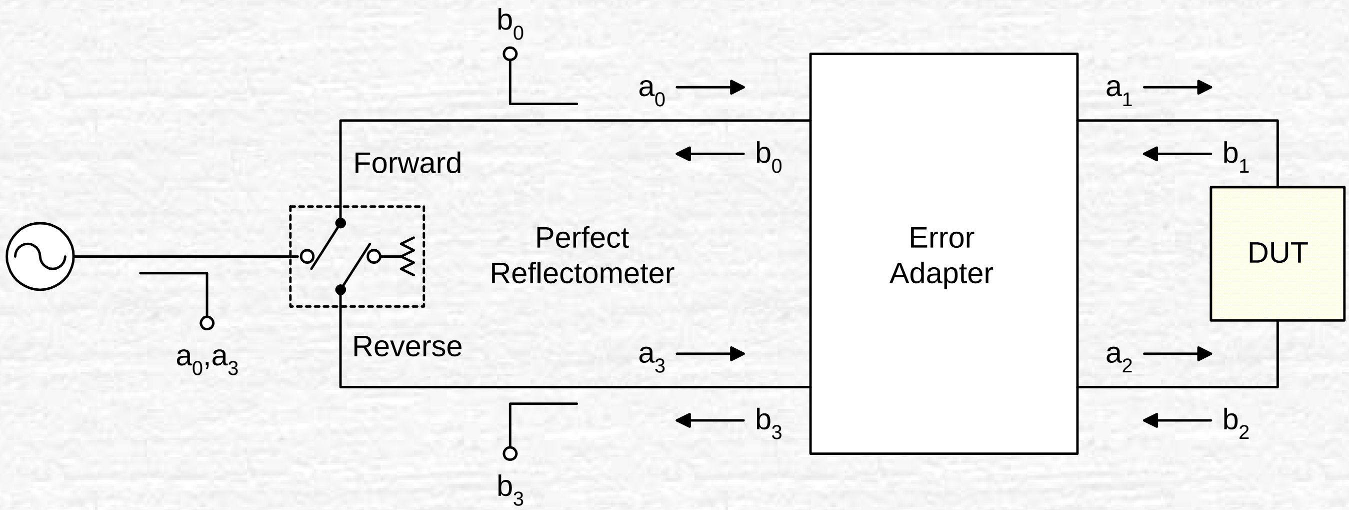
12-Term Method

16-Term Method

8-Term Method

The classic 12-term model will be developed first. Then the more recent 16-term and 8-term models will be described.

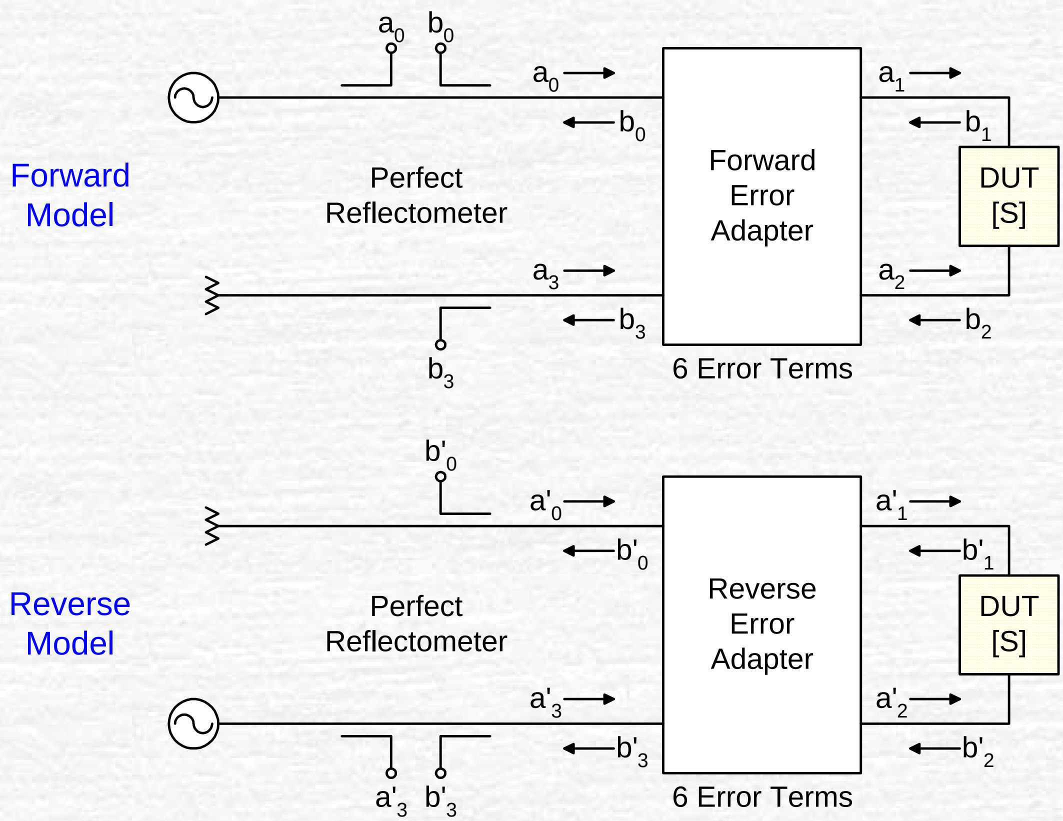
12-Term Error Model



Many of the older and lower cost network analyzers use three couplers instead of four for two port measurements. This puts the switch between the incident and reflected couplers. If the switch characteristics change as the switch is flipped from forward to reverse, the error adapter's error terms will change. Even some four coupler network analyzers use this method as well, choosing not to use the fourth coupler for all measurements.

This is the original technique uses in the 1960's for the first automatic network analyzers. This TOSL (through, open, short, load) 12-term model has been used for many years and is still widely used today.

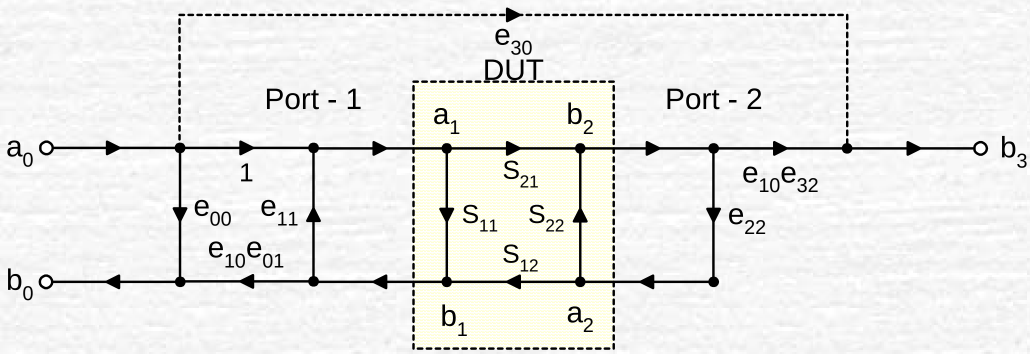
12-Term Error Model



The two-port case can be modeled in the same manner as the one-port. A fictitious error adapter is placed between the two-port DUT and the ‘perfect reflectometer’ measurement ports. This error adapter contains the 6 error terms for the forward direction. A similar 6 term model is used in the reverse direction.

12-Term Error Model

FORWARD MODEL



e_{00} = Directivity

e_{11} = Port-1 Match

$(e_{10}e_{01})$ = Reflection Tracking

$(e_{10}e_{32})$ = Transmission Tracking

e_{22} = Port-2 Match

e_{30} = Leakage

$$S_{11M} = \frac{b_0}{a_0} = e_{00} + (e_{10}e_{01})\frac{S_{11} - e_{22}\,\Delta_S}{1 - e_{11}S_{11} - e_{22}S_{22} + e_{11}e_{22}\,\Delta_S}$$

$$S_{21M} = \frac{b_3}{a_0} = e_{30} + (e_{10}e_{32})\frac{S_{21}}{1 - e_{11}S_{11} - e_{22}S_{22} + e_{11}e_{22}\,\Delta_S}$$

$$\Delta_S = S_{11}S_{22} - S_{21}S_{12}$$

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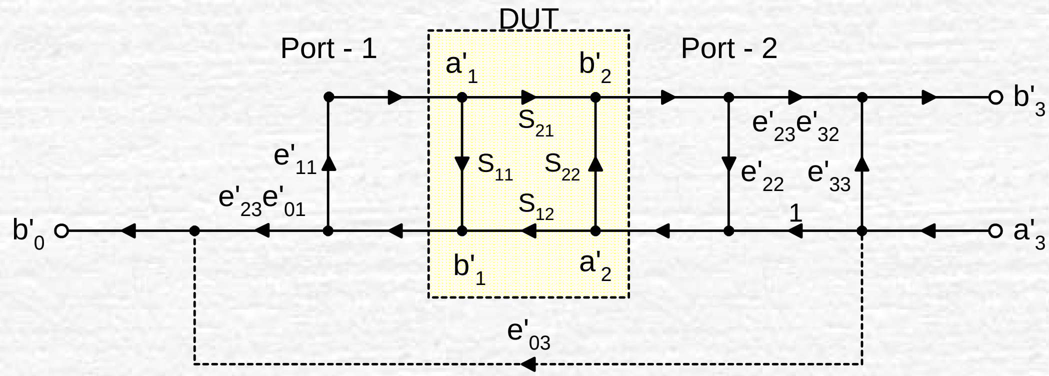
Solving the forward flow graph yields measurements S_{11M} and S_{21M}. These two equations contain all four actual S-parameters of the DUT and the six forward error terms.

12-Term Error Model

REVERSE MODEL



$e'_{33} =$ Directivity

$e'_{11} =$ Port-1 Match

$(e'_{23}e'_{32}) =$ Reflection Tracking

$(e'_{23}e'_{01}) =$ Transmission Tracking

$e'_{22} =$ Port-2 Match

$e'_{03} =$ Leakage

$$S_{22M} = \frac{b'_3}{a'_3} = e'_{33} + (e'_{23}e'_{32}) \frac{S_{22} - e'_{11}\Delta_S}{1 - e'_{11}S_{11} - e'_{22}S_{22} + e'_{11}e'_{22}\Delta_S}$$

$$S_{12M} = \frac{b'_0}{a'_3} = e'_{03} + (e'_{23}e'_{01}) \frac{S_{12}}{1 - e'_{11}S_{11} - e'_{22}S_{22} + e'_{11}e'_{22}\Delta_S}$$

$$\Delta_S = S_{11}S_{22} - S_{21}S_{12}$$

Solving the reverse flow graph yields measurements S_{22M} and S_{12M}. These two equations contain all four actual S-parameters of the DUT and the six reverse error terms.

The forward and reverse equations combine to give four equations containing the four actual S-parameters of the DUT and 12 error terms. If the 12 error terms are known these four equations can be solved for the actual S-parameters of the DUT.

12-Term Error Model

$$S_{11} = \frac{\left(\frac{S_{11M} - e_{00}}{e_{10}e_{01}}\right)\left[1 + \left(\frac{S_{22M} - e'_{33}}{e'_{23}e'_{32}}\right)e'_{22}\right] - e_{22}\left(\frac{S_{21M} - e_{30}}{e_{10}e_{32}}\right)\left(\frac{S_{12M} - e'_{03}}{e'_{23}e'_{01}}\right)}{D}$$

$$S_{21} = \frac{\left(\frac{S_{21M} - e_{30}}{e_{10}e_{32}}\right)\left[1 + \left(\frac{S_{22M} - e'_{33}}{e'_{23}e'_{32}}\right)(e'_{22} - e_{22})\right]}{D}$$

$$S_{22} = \frac{\left(\frac{S_{22M} - e'_{33}}{e'_{23}e'_{32}}\right)\left[1 + \left(\frac{S_{11M} - e_{00}}{e_{10}e_{01}}\right)e_{11}\right] - e'_{11}\left(\frac{S_{21M} - e_{30}}{e_{10}e_{32}}\right)\left(\frac{S_{12M} - e'_{03}}{e'_{23}e'_{01}}\right)}{D}$$

$$S_{12} = \frac{\left(\frac{S_{12M} - e'_{03}}{e'_{23}e'_{01}}\right)\left[1 + \left(\frac{S_{11M} - e_{00}}{e_{10}e_{01}}\right)(e_{11} - e'_{11})\right]}{D}$$

$$D = \left[1 + \left(\frac{S_{11M} - e_{00}}{e_{10}e_{01}}\right)e_{11}\right]\left[1 + \left(\frac{S_{22M} - e'_{33}}{e'_{23}e'_{32}}\right)e'_{22}\right] - \left(\frac{S_{21M} - e_{30}}{e_{10}e_{32}}\right)\left(\frac{S_{12M} - e'_{03}}{e'_{23}e'_{01}}\right)e_{22}e'_{11}$$

This is the result of solving the four simultaneous measured s-parameter equations. Note that each actual s-parameter calculated requires measuring all four S-parameters as well as knowing the 12 error terms.

12-Term Error Model

Calibration

STEP 1: Calibrate Port-1 using One-Port procedure

Solve for e_{11}, e_{00}, & $(e_{10}e_{01})$, Calculate $(e_{10}e_{01})$ from Δ_e

STEP 2: Connect Z_0 terminations to Ports 1 & 2

Measure S_{21M} gives e_{30} directly

STEP 3: Connect Ports 1 & 2 together

$$e_{22} = \frac{S_{11M} - e_{00}}{S_{11M}e_{11} - \Delta_e}$$

$$e_{10}e_{32} = (S_{21M} - e_{30})(1 - e_{11}e_{22})$$

Use the same process for the reverse model

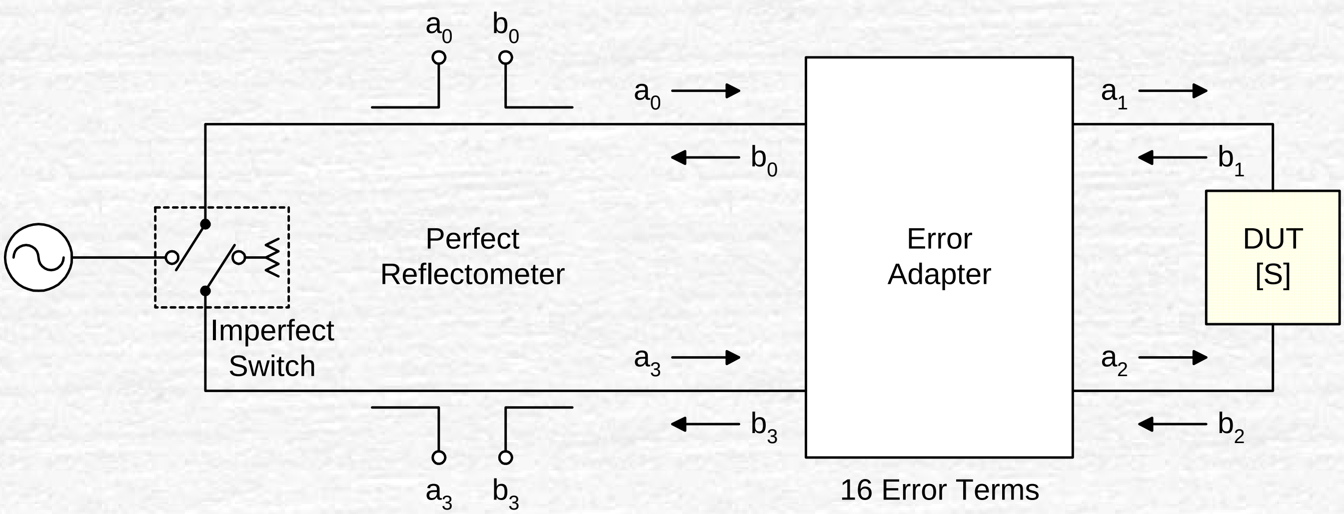
The 12 error terms will now be determined. First solve for the 6 terms in the forward direction. Then the same procedure can be used to solve for the 6 reverse terms.

Step one calibrates port-1 of the network analyzer using the same procedure used in the one-port case. This determines the directivity, match, and reflection tracking at port-1 (e_{00}, e_{11}, and Δ_e). From Δ_e the reflection tracking $(e_{10}e_{01})$ can be calculated.

Step two measures the leakage or crosstalk error (e_{30}) from port-1 to port-2 directly by placing loads on each of the ports.

Step three consists of connecting port-1 and port-2 together. Then measure the port-2 match (e_{22}) directly with the calibrated port-1 reflectometer. Then with the ports connected, measure the transmitted signal and calculate the transmission tracking ($e_{10}e_{32}$).

16-Term Error Model



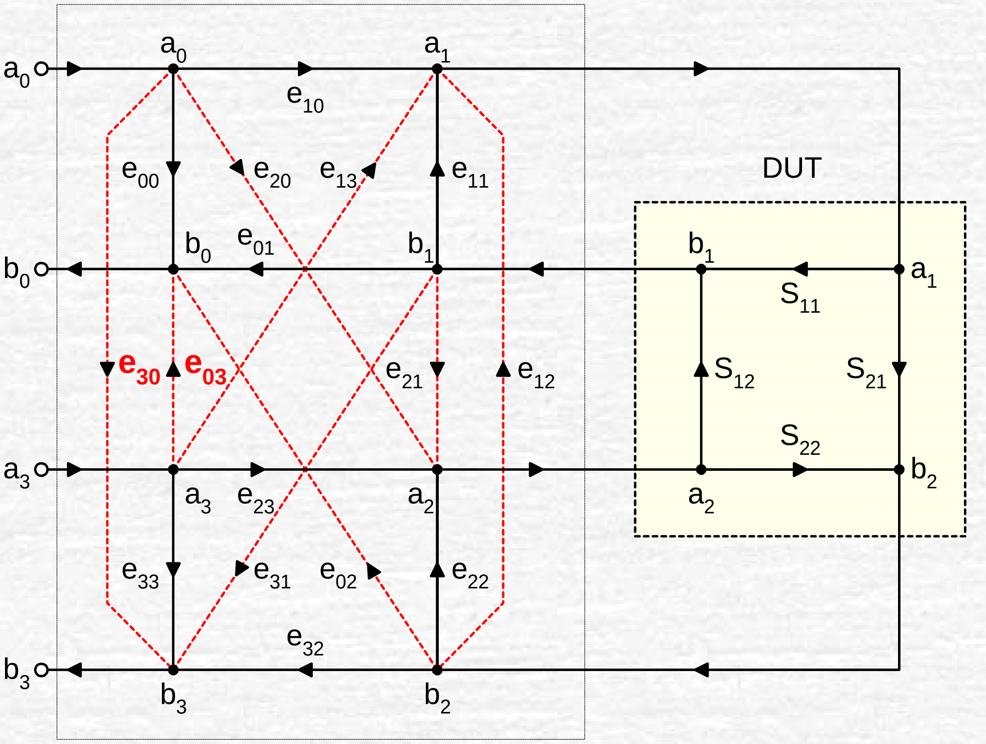
To remove the effects of an imperfect switch, use the procedure described later.

The complete system can be modeled using an error adapter between the DUT and the actual measurements. This error adapter describes all the possible stationary and linear errors that are caused by the system. Since the adapter is a 4-port it contains 16 error terms. If these error terms can be determined then the DUT S-parameters can be calculated from the measured S-parameters.

When making ratio measurements one of the terms can be normalized to 1 leaving 15 error terms.

When using 4 couplers placed after the switch the errors of the switch can be removed and will be discussed later.

16-Term Error Model



e_{00}, e_{33} Directivity
e_{11}, e_{22} Port Match
e_{10}, e_{01}, e_{32}, e_{23} Tracking
e_{30}, e_{03} Primary Leakage

All others are lower level leakage paths

One of the 16 error terms can be normalized to yield 15 error terms

There are 16 terms of which 8 are critical. These 8 include the directivity, port match and tracking errors.

The leakage terms add 8 additional error terms to the model. Not only is the traditional crosstalk term included, but switch leakage, signals reflecting from the DUT and leaking to the transmission port, common mode inductance, and other leakage paths are included. In a coaxial or waveguide system, assuming the switch has high isolation, these errors are small. But in a fixtured or wafer probe environment these errors can be much larger.

In a wafer probe environment it is important that the error terms do not change as the probes are moved around the circuit. If the error terms change, the 16-term model then changes and the accuracy will reduce.

Again the error terms can be normalized so that for ratio measurements there are only 15 error terms.

16-Term Error Model

Error Model

$$\begin{bmatrix} b_0 \\ b_3 \\ \hline a_0 \\ a_3 \end{bmatrix} = \begin{bmatrix} \mathbf{T_1} & \mathbf{T_2} \\ \hline \mathbf{T_3} & \mathbf{T_4} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \hline b_1 \\ b_2 \end{bmatrix}$$

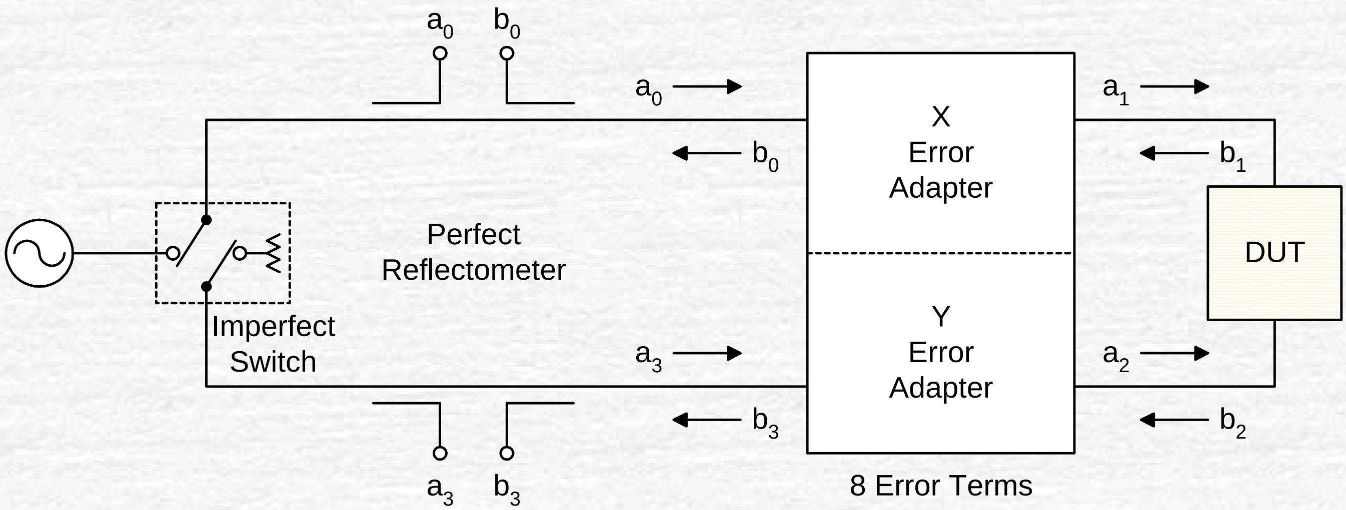
Measured S-Parameters	$\mathbf{S_M = (T_1S + T_2)(T_3S + T_4)^{-1}}$
Actual S-Parameters	$\mathbf{S = (T_1 - S_MT_3)^{-1}(S_MT_4 - T_2)}$
Linear-in-T Form	$\mathbf{T_1S + T_2 - S_MT_3S - S_MT_4 = 0}$

With 15 or more independent observations the linear matrix equation can be solved. TRL as well as TOSL calibration methods are possible.

One method of solving this system is to use T-parameters (cascade parameters) that relate the measured waves to the DUT waves. This system is solved by partitioning the T matrix into 4 2x2 matrices and then solving the matrix equations which yields the bi-linear form above. This allows us to form a linear set of equations that can be solved for the error terms.

By using enough known standards, a set of linear equations is created that can be solved for the individual error terms. At least 5 two port standards are required to solve the system of equations. However, most of these can be degenerate two port standards that involve combinations of opens, shorts, loads and transmission lines.

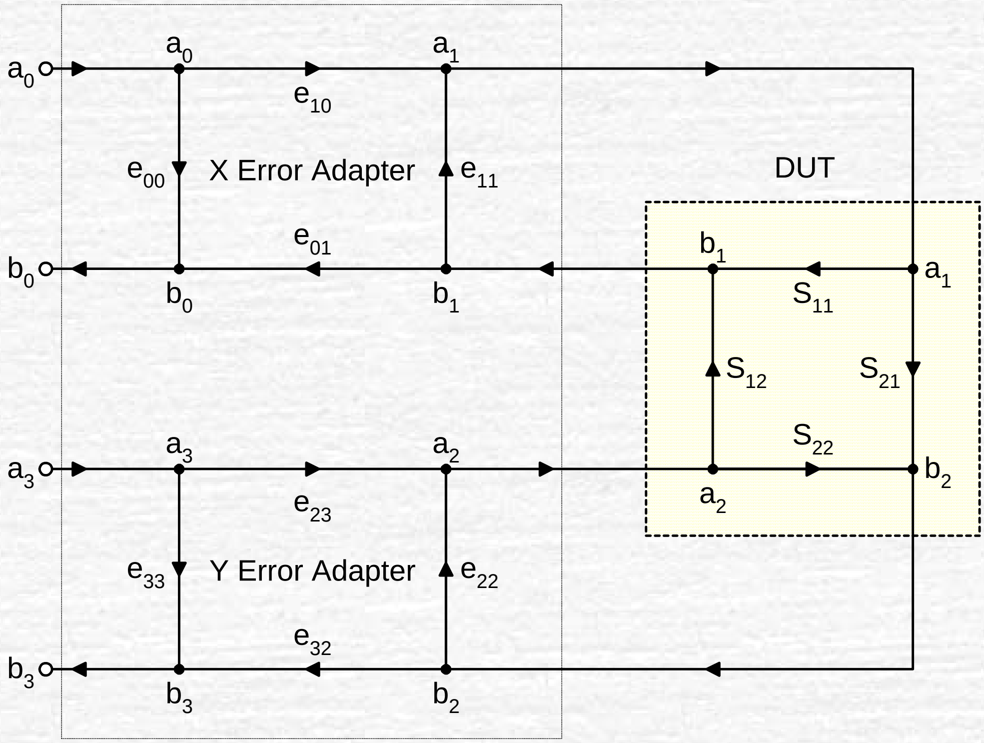
8-Term Error Model



To remove the effects of an imperfect switch, use the procedure described later.

The 8-term model can be derived from the 16-term model. First assume that the leakage terms are all zero. Or that the two primary leakage terms can be determined in a separate calibration step. Then assume that the switch is perfect and does not change the port match of the network analyzer as it is switched from forward to reverse. This assumption is valid if there are 4 measurement channels that are all on the DUT side of the switch. Then it is possible to mathematically ratio out the switch. This mathematical approach will be discussed later.

8-Term Error Model



One of the 8 error terms can be normalized to yield 7 error terms

The flow graph consists of an error adapter at the input and output of the DUT. For ratio measurements of S-parameters, the number of error terms is reduced to 7 since the error terms can be normalized.

8-Term Error Model

$$\begin{bmatrix} b_0 \\ b_3 \\ \hline a_0 \\ a_3 \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{T_1} & \mathbf{T_2} \\ \hline \mathbf{T_3} & \mathbf{T_4} \end{array}\right] \begin{bmatrix} a_1 \\ a_2 \\ \hline b_1 \\ b_2 \end{bmatrix}$$

$$\mathbf{T_1} = \begin{bmatrix} -\Delta_X & 0 \\ 0 & -k\Delta_Y \end{bmatrix} \qquad \mathbf{T_2} = \begin{bmatrix} e_{00} & 0 \\ 0 & ke_{33} \end{bmatrix}$$

$$\mathbf{T_3} = \begin{bmatrix} -e_{11} & 0 \\ 0 & -ke_{22} \end{bmatrix} \qquad \mathbf{T_4} = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$$

$$k = \frac{e_{10}}{e_{23}}, \quad \Delta_X = e_{00}e_{11} - e_{10}e_{01}, \quad \Delta_Y = e_{22}e_{33} - e_{32}e_{23}$$

This is one of many possible mathematical formulation for the 8-term error model. Consider the error adapter as just one adapter between the perfect measurement system and the DUT. Then model this error adapter using the cascade T-parameters. This T-parameter matrix (T) can be partitioned, as described in the 16-term method, into the four diagonal sub matrixes T_1, T_2, T_3, and T_4. The 7 error terms are now defined as Δ_X, $k\Delta_Y$, e_{00}, ke_{33}, e_{11}, ke_{22}, and k. Which are the tracking, directivity, and match errors.

8-Term Error Model

Measured S-Parameters	$\mathbf{S_M = (T_1S + T_2)(T_3S + T_4)^{-1}}$
Actual S-Parameters	$\mathbf{S = (T_1 - S_MT_3)^{-1}(S_MT_4 - T_2)}$
Linear-in-T Form	$\mathbf{T_1S + T_2 - S_MT_3S - S_MT_4 = 0}$

Expanding Yields:

e_{00}	$+ S_{11}S_{11M}e_{11}$	$- S_{11}\Delta_X$	$+ 0$	$+ S_{21}S_{12M}(ke_{22})$	$+ 0$	$+ 0$	$= S_{11M}$
0	$+ S_{12}S_{11M}e_{11}$	$- S_{12}\Delta_X$	$+ 0$	$+ S_{22}S_{12M}(ke_{22})$	$+ 0$	$- S_{12M}k$	$= 0$
0	$+ S_{11}S_{21M}e_{11}$	$+ 0$	$+ 0$	$+ S_{21}S_{22M}(ke_{22})$	$- S_{21}(k\Delta_Y)$	$+ 0$	$= S_{21M}$
0	$+ S_{12}S_{21M}e_{11}$	$+ 0$	$+ (ke_{33})$	$+ S_{22}S_{22M}(ke_{22})$	$- S_{22}(k\Delta_Y)$	$- S_{22M}k$	$= 0$

Using this approach the measured S-parameters formulation is a 'bilinear matrix equation.' The equation can be 'inverted' to solve for the actual S-parameters. And most important the relationship can be put in linear form. Expanding this matrix equation for the two-port case yields 4 equations with 4 measured S-parameters, 4 actual S-parameters, and 7 error terms. Note that these 4 equations are linear with regards to the 7 error terms.

This approach is particularly attractive for multi-port measurement systems. The matrix formulation does not change as additional ports are added.

8-Term Error Model

Using the cascade parameters in matrix form yields

MEASURED

$$\mathbf{T_M = T_X T\, T_Y}$$

$$\mathbf{T} = \frac{1}{S_{21}}\begin{bmatrix} -\Delta_S & S_{11} \\ -S_{22} & 1 \end{bmatrix}$$

$$\Delta_S = S_{11}S_{22} - S_{12}S_{21}$$

$$\mathbf{T_X} = \frac{1}{e_{10}}\begin{bmatrix} -\Delta_X & e_{00} \\ -e_{11} & 1 \end{bmatrix}$$

$$\Delta_X = e_{00}e_{11} - e_{10}e_{01}$$

ACTUAL

$$\mathbf{T = T_X^{-1} T_M\, T_Y^{-1}}$$

$$\mathbf{T_M} = \frac{1}{S_{21M}}\begin{bmatrix} -\Delta_M & S_{11M} \\ -S_{22M} & 1 \end{bmatrix}$$

$$\Delta_M = S_{11M}S_{22M} - S_{12M}S_{21M}$$

$$\mathbf{T_Y} = \frac{1}{e_{32}}\begin{bmatrix} -\Delta_Y & e_{22} \\ -e_{33} & 1 \end{bmatrix}$$

$$\Delta_Y = e_{22}e_{33} - e_{32}e_{23}$$

$$\mathbf{T_M} = \frac{1}{(e_{10}e_{32})}\begin{bmatrix} -\Delta_X & e_{00} \\ -e_{11} & 1 \end{bmatrix}\mathbf{T}\begin{bmatrix} -\Delta_Y & e_{22} \\ -e_{33} & 1 \end{bmatrix} = \frac{1}{(e_{10}e_{32})}\mathbf{ATB}$$

Another approach is to cascade of the input error box (X), the DUT, and the output error box (Y). The measured result of this cascade is most easily calculated by using the cascade matrix definition (T-parameters).

This formulation was used by Engen and Hoer in their classic TRL development for the six-port network analyzer. And is the same approach used in the HP 8510 network analyzer.

From the last equation in the slide above, the 7 error terms are easily identified. There are 3 at port-1 (Δ_X, e_{00}, and e_{11}) and 3 at port-2 (Δ_Y, e_{22}, and e_{33}) and one transmission term ($e_{10}e_{32}$).

The calibration approach require enough calibration standards to allow at least 7 independent observations of the measurement system.

8-Term Calibration Examples

Seven or more independent known conditions must be measured
A known impedance (Z_0) and a port-1 to port-2 connection are required

TRL & LRL	Thru (T) or Line (L) with known S-parameters [4 conditions]	Unknown equal Reflect (R) on port-1 and port-2 [1 condition]	Line (L) with known S_{11} and S_{22} [2 conditions]
TRM & LRM	Thru (T) or Line (L) with known S-parameters [4 conditions]	Unknown equal Reflect (R) on port-1 and port-2 [1 condition]	Known Match (M) on port-1 and port-2 [2 conditions]
TXYZ & LXYZ	Thru (T) or Line (L) with known S-parameters [4 conditions]	3 known Reflects (XYZ) on port-1 or port-2 [3 conditions]	
Traditional TOSL (Overdetermined)	Thru (T) with known S-parameters [4 conditions]	3 known Reflects (OSL) on port-1 [3 conditions]	3 known Reflect (OSL) on port-2 [3 condition]
LRRM	Line (L) with known S-parameters [4 conditions]	2 unknown equal Reflects (RR) on port-1 and port-2 [2 conditions]	Known match (M) on port-1 [1 condition]
UXYZ	Unknown Line (U) with $S_{12} = S_{21}$ [1 condition]	3 known Reflects (XYZ) on port-1 [3 conditions]	3 known Reflects (XYZ) on port-2 [3 conditions]

There is a number of calibration techniques that have been developed based on the 8-term error model. Seven or more independent conditions must be measured. There must be a known impedance termination or a known transmission line. And port-1 and port-2 must be connected for one of the measurements.

The list of calibration approaches can be much longer than the ones shown above. And there continues to be new and novel ways to solve for the seven error terms and calibrate the system.

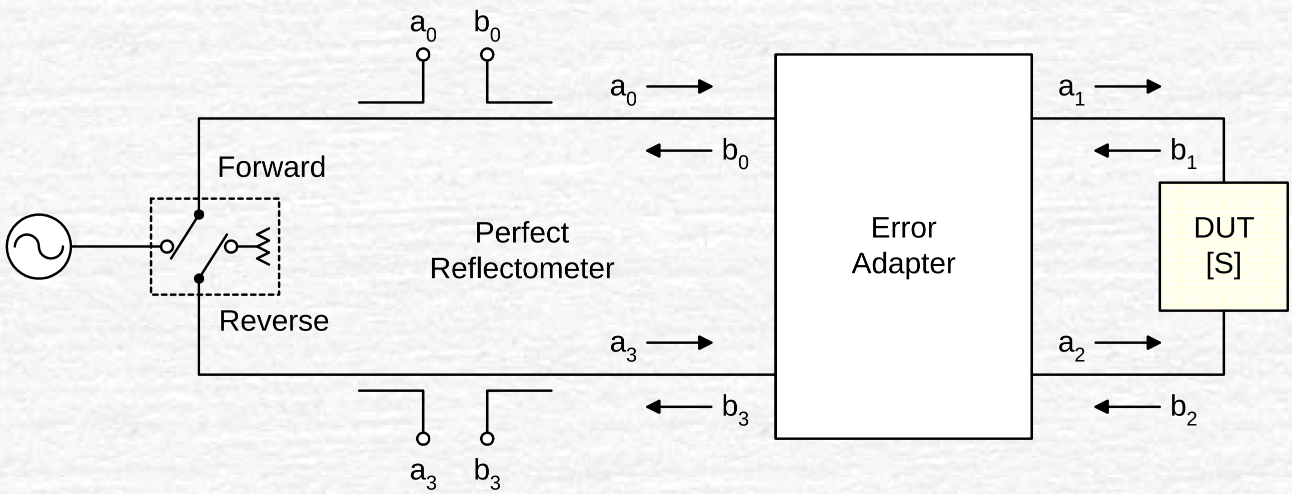
The 8-term error model approach has yielded more accurate calibration methods as well as simplified the calibration process. TRL and LRL provide the best accuracy. The other methods simplify the calibration steps compared to the older TOSL 12 term model. In one case (UXYZ above) the thru standard does not need to be known as long as it is reciprocal.

Measuring

S-parameters

A method for measuring s-parameters without terminating the ports with a Z_0 load will now be described.

Measuring S-parameters



<u>Forward</u>

$$b_0 = S_{11M}a_0 + S_{12M}a_3$$
$$b_3 = S_{21M}a_0 + S_{22M}a_3$$

<u>Reverse</u>

$$b'_0 = S_{11M}a'_0 + S_{12M}a'_3$$
$$b'_3 = S_{21M}a'_0 + S_{22M}a'_3$$

Normally we measure S-parameters by terminating one port with a Z_0 impedance and then measure the input reflection and transmission coefficients then flip the switch and make the same set of measurements in the reverse direction. However, with 4 couplers we can measure both the forward and reverse terminating impedances of the switch. This allows us to "ratio out" or remove the effects of the imperfect switch.

Note that the measurement channels are all on the DUT side of the switch. This allows the measurements of the incident and reflected signals at both outputs of the switch and the switch match errors can be calculated.

Mathematically the S-parameters of the system generate 4 equations. 2 in the forward direction and 2 in the reverse direction. These 4 equations can then be solved for the 4 measured S-parameters. This general way of measuring S-parameters does not require the DUT to be terminated in a Z_0 environment.

Measuring S-parameters

By defining

$$\Gamma_1 = \frac{a_0'}{b_0'} \quad \text{and} \quad \Gamma_2 = \frac{a_3}{b_3}$$

$$S_{11M} = \frac{\frac{b_0}{a_0} - \frac{b'_0}{a'_3}\frac{b_3}{a_0}\Gamma_2}{d} \qquad S_{12M} = \frac{\frac{b'_0}{a'_3} - \frac{b_0}{a_0}\frac{b'_0}{a'_3}\Gamma_1}{d}$$

$$S_{21M} = \frac{\frac{b_3}{a_0} - \frac{b'_3}{a'_3}\frac{b_3}{a_0}\Gamma_2}{d} \qquad S_{22M} = \frac{\frac{b'_3}{a'_3} - \frac{b_3}{a_0}\frac{b'_0}{a'_3}\Gamma_1}{d}$$

$$d = 1 - \frac{b_3}{a_0}\frac{b'_0}{a'_3}\Gamma_1\Gamma_2$$

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Solving the 4 previous equations yield the above results. Note that the equations are written to allow ratio measurements by the network analyzer. Typically the network analyzer is more accurate making measurements this way. Noise and other common mode errors are reduced.

Using this method for measuring the 4 S-parameters requires 6 ratio measurements. The additional two measurements are required to remove the effects of the switch. However, these two additional measurements (Γ_1 and Γ_2) need only be made during the calibration step since they do not change during measurement assuming the switch is stable.

Accuracy

of

Error-Correction

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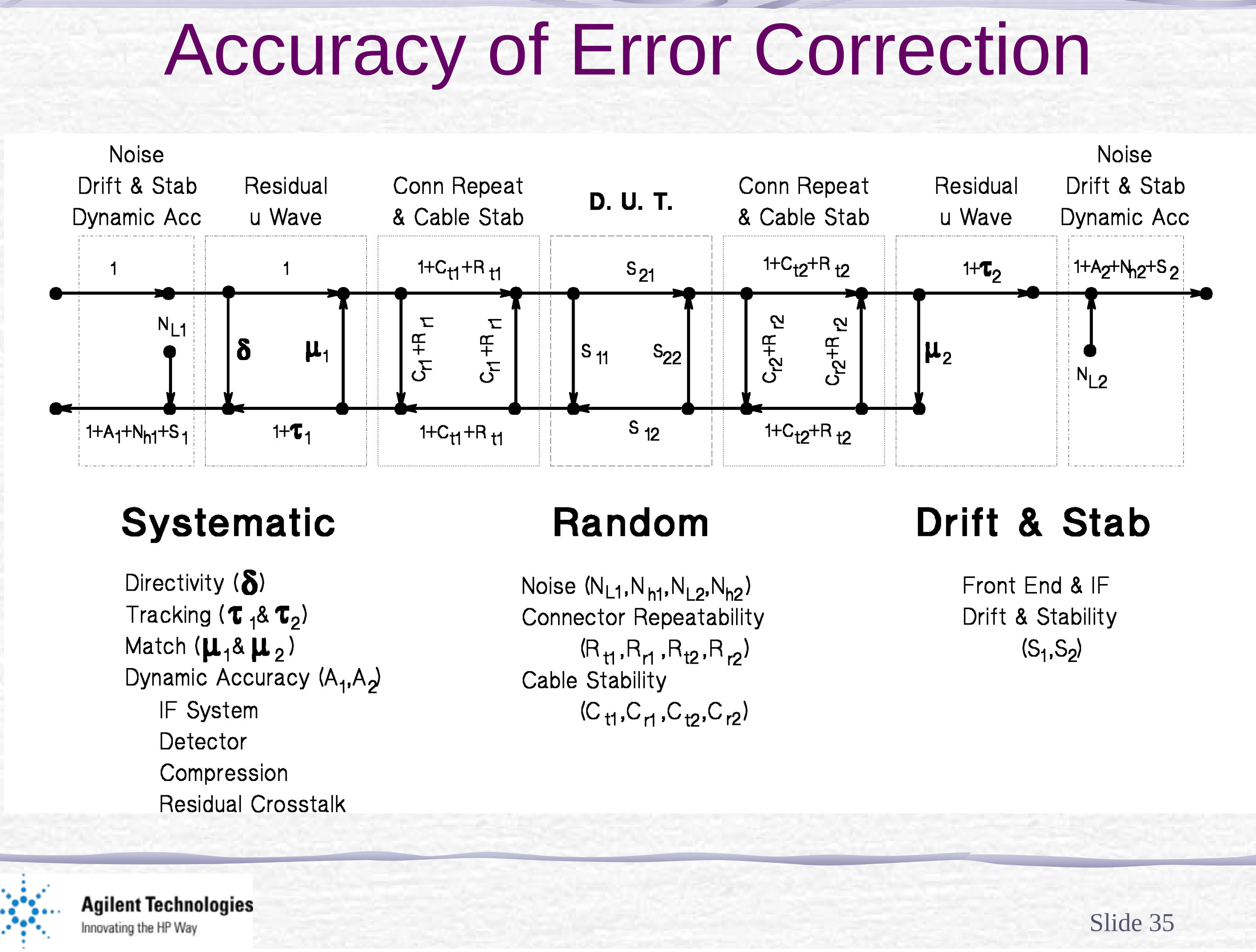
The accuracy of error corrected measurements will now be briefly discussed.



The error corrected measurement system can be nicely described using flow graphs. The flow graph after error correction is very similar to the one before correction but with the error terms reduced.

The device under test will be degraded by the following residual errors and hardware imperfections. The residual microwave errors (δ, $\tau_{1,2}$ and $\mu_{1,2}$) are mainly determined by the errors of the calibration standards and are respectively the residual directivity, residual tracking and residual match. Subscript 1 applies to port-1 and subscript 2 applies to port-2.

$C_{t1,t2}$ describes the cable transmission coefficient change and $C_{r1,r2}$ describe the change in the cable reflection coefficient. $R_{t1,t2}$ characterizes the connector transmission repeatability error and $R_{r1,r2}$ characterizes the connector reflection repeatability error. The low level noise ($N_{L1,2}$) of the converter determines the sensitivity of the system, and a high level noise of the LO and IF ($N_{h1,2}$) contribute to the trace noise on the measurement data.

The front end and IF hardware will drift with time and temperature as characterized by the stability terms ($S_{1,2}$). The nonlinearities of the system with measurement level are described by the dynamic accuracy ($A_{1,2}$).

Accuracy of Error Correction

UNCERTAINTY EQUATIONS

Reflection Magnitude Uncertainity

$$\Delta S_{11} = \text{Systematic} + [\text{Random}^2 + (\text{Drift \& Stab})^2]^{1/2}$$

$$\text{Systematic} = \delta + \tau_1 S_{11} + \mu_1 S_{11}^2 + S_{21} S_{12} \mu_2 + A_1 S_{11}$$

Transmission Magnitude Uncertainty

$$\Delta S_{21} = \text{Systematic} + [\text{Random}^2 + (\text{Drift \& Stab})^2]^{1/2}$$

$$\text{Systematic} = (\mu_1 S_{11} + \mu_2 S_{22} + \mu_1 \mu_2 S_{21} S_{12} + \tau_2 + A_2)\, S_{21}$$

The flow graph can be solved and simplified to show the total system uncertainty. These equations calculate the magnitude uncertainty (ΔS_{11} and ΔS_{21}) with each error term defined by its absolute magnitude. The first part of the equation describes the systematic errors and these errors typically add up in a worst case manner. The random, drift and stability errors are typically characterized in an RSS fashion in the second part. These equations do not include second order effects.

Accuracy of Error Correction

Approximate Residual Microwave Errors

$$\delta = -\mu_2 = -\Delta_L$$
$$\tau_1 = \Delta_S/2 - \Delta_O/2$$
$$\mu_1 = \Delta_L - \Delta_S/2 - \Delta_O/2$$
$$\tau_2 = M_1\mu_2 + M_2\mu_1$$

δ = Residual directivity
μ_1 and μ_2 = Residual port match
τ_1 and τ_2 = Residual tracking
M_1 and M_2 = Raw uncorrected port match

Δ_L = Error of the Load, match or line standard
Δ_O = Error of the Open Standard (0 for TRL)
Δ_S = Error of the short standard (0 for TRL)

The reflection residual microwave errors are mainly determined by the quality of the calibration standards. If the assumed values of the standards and their measured value are known, then the residual errors can be calculated. These approximate residual errors only apply to TOSL and TRL type calibrations.

Accuracy of Error Correction

APC-7 (7 mm Coax) at 18 GHz

Residual Errors	OSL Fixed Load	OSL Sliding Load	TRL	TRM
Directivity δ	-40 dB	-52 dB	-60 dB	-40 dB
Match μ	-35 dB	-41 dB	-60 dB	-40 dB
Reflection Tracking τ	± .1 dB	± .05 dB	± .01 dB	± .01 dB

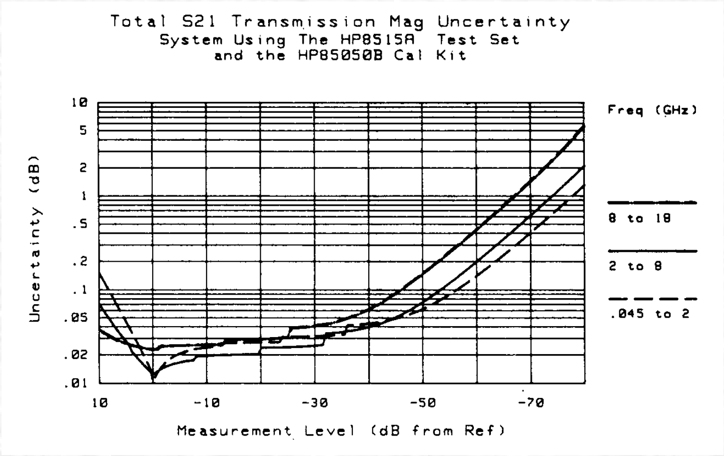
his table gives the tradeoff in accuracy for various coax calibration methods. The example is for PC-7 mm but can be scaled to other connector types. The relative differences stay about the sam he OSL (Fixed Load) cal is the least expensive and easy to use. The tradeoff is that it has the west accuracy, but this may be fine for many measurements. The directivity error is determined the load. The match and tracking errors are mainly determined by the the phase error of the pen.

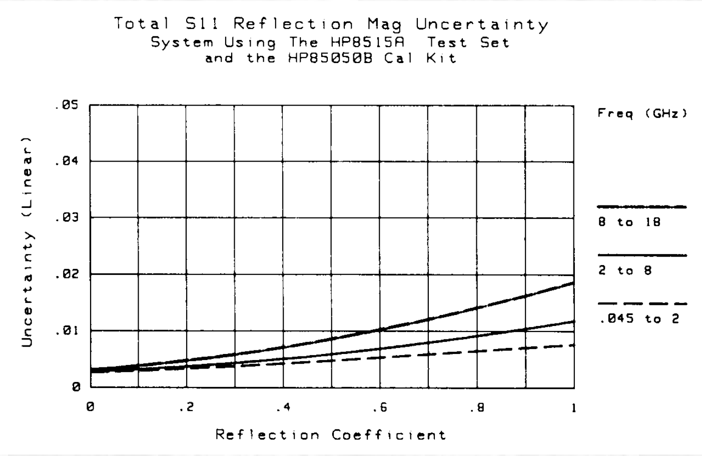
he OSL (Sliding Load) is the traditional calibration that has been used for many years for accura easurements. It is fairly expensive and sometimes the sliding load is not as easy to use. OSL librations are popular for one-port measurements.

ith the OSL calibrations the short and open determine most of the match and tracking error and is error does not change dramatically with improved directivity values.

he TRL method provides the best accuracy and particularly for the match and tracking terms. It i ot always easy to use and the precision lines are reasonably expensive. The TRM calibration doe ot change the directivity error over the OSL (load-fixed) method but reduces the match and acking errors. The TRL and TRM calibration methods need a two port measurement system in der to calibrate. The TRM calibration is the easiest to use.

Accuracy of Error Correction





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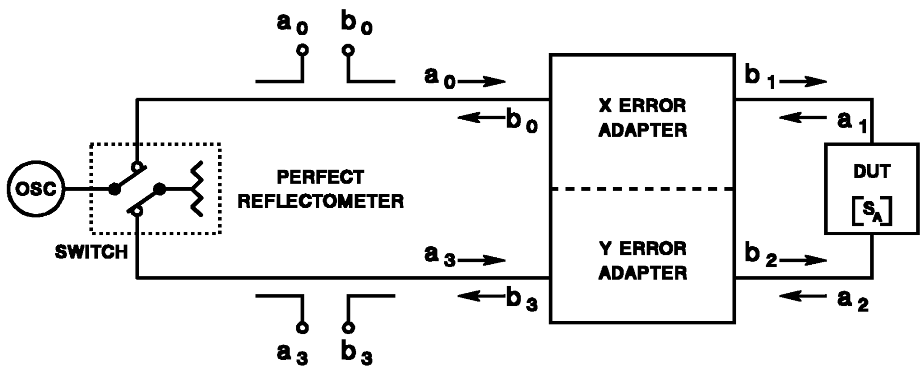
The first graph shows the transmission accuracy after error correction using APC-7 connectors. The main error at high levels is due to compression. At low levels it is primarily due to noise and uncorrected leakage. This graph assumes that S_{11} and S_{22} of the DUT are zero.

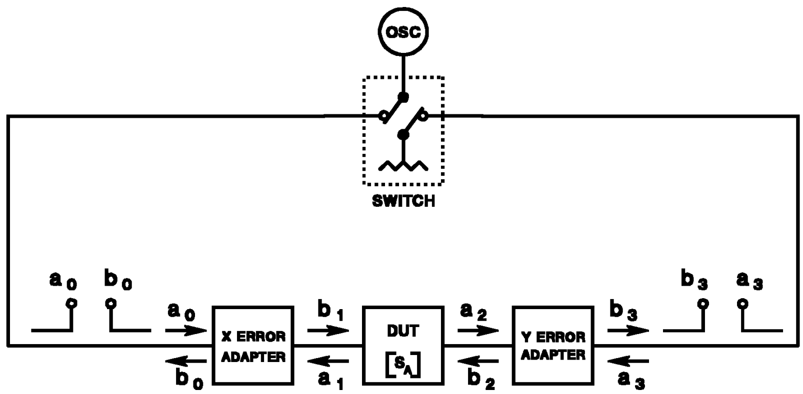
The APC-7 reflection magnitude accuracy using the sliding load calibration is shown in the second graph. The main error at small reflection coefficients is due to the sliding load residual match. At high levels the sliding load match and the open circuit phase error are the primary contributors. This graph assumes S_{12} of the DUT is zero.

Appendix

Example: TRL



After removing leakage terms 8 error terms remain



The best know calibration method using the 8-term model is TRL. We will now review this calibration method. The math nomenclature is slightly different in this review.

The first step involves separating the system into a perfect reflectometer followed by a 4-port error adapter. This error adapter represents all the errors in the system that can be corrected. It can be split into two 2-port error adapters, X (at port-1) and Y (at port-2), after removing the leakage (crosstalk) terms as a first step in the calibration. Since X and Y are 2-ports it would appear there are 8 unknowns to find, however since all measurements are made as ratios of the b's and a's, there are actually only 7 error terms to calculate. This means that only 7 characteristics of the calibration standards are required to be known. If a thru (4 known characteristics) is used as one of the standards, only 3 additional characteristics of the standards are needed.

Example: TRL

(1) $M = X\ A\ Y$, measured DUT

(2) $M_1 = X\ C_1\ Y$, measured 2-port cal std #1

(3) $M_2 = X\ C_2\ Y$, measured 2-port cal std #2

(4) $M_3 = X\ C_3\ Y$, measured 2-port cal std #3

It is convenient to use T-parameters because it allows one to represent the overall measurement, M, of the DUT, A, as corrupted by the error adapters as a simple product of the matrixes,

$$M = XAY.$$

In a similar manner, each measurement of three 2-port standards, C_1, C_2, and C_3 can be represented as M_1, M_2, and M_3.

$$M_1 = XC_1Y$$

$$M_2 = XC_2Y$$

$$M_3 = XC_3Y$$

Example: TRL

Measurements of the 3 two-port standards yields 12 independent equations.

Only 7 equations are needed to calibrate the system.

Equations (2), (3), and (4) can be solved for X.

Also 5 terms of the three two-port calibration standards can be determined.

While there are 7 unknowns, measuring three 2-port standards yields a set of 12 equations. Due to this redundancy, it is not necessary to know all the parameters of all the standards. X and Y can be solved for directly plus 5 characteristics of the calibration standards.

Example: TRL

C_1 **Must be totally known.**

C_2 **Can have 2 unknown transmission terms.**
The 2 reflection coef must be known.

C_3 **Can have 3 unknowns.**
If $S_{11} = S_{22}$, no other terms are needed.
Best if highly reflective.

The standards must be independent from each other.

All 4 parameters of C_1 must be known but only 2 parameters for C_2 and one for C_3 if $S_{11} = S_{22}$. The simplest of all known standards is a through line, so let C_1 be a thru, C_2 a Z_0 matched load or line with $S_{12} = S_{21}$ and C_3 a high reflect with $S_{11}=S_{22}$. If needed, impedance renormalization can be used to shift to a different impedance base. The other parameters of C_2 and C_3 can be solved from the calibration process.

For this calibration method there are several combinations of standards that fit the requirements. However, there are also choices that generate ill-conditioned solutions or singularities. In choosing appropriate standards, one standard needs to be Z_0 based, one needs to present a high mismatch reflection, and one needs to connect port-1 to port-2. In addition, all three standards need to be sufficiently different to create three independent measurements.

Example: TRL

Possible Combinations of Two-Port Standards
Must know 7 characteristics

Cal Type	Std C_1 [S]	Std C_3 $S_{11}=S_{22}$	Std C_2 S_{11} & S_{22}
TRL	Thru	Reflect	Line
TRM	Thru	Reflect	Match
TRA	Thru	Reflect	Attenuator
LRL	Line	Reflect	Line
LRM	Line	Reflect	Match
LRA	Line	Reflect	Attenuator

There are several possible strategies in choosing standards. For the first standard (C_1), the use of a zero length thru is an obvious selection. But a non-zero length thru is also acceptable if its characteristics are known or the desired reference plane is in the center of the non-zero length thru. This standard will determine 4 of the error terms.

The second standard (C_2) needs to provide a Z_0 reference. In this solution, only the match of this standard needs to be of concern. Its S_{21} and S_{12} can be any value and do not need to be known. In fact, they will be found during the calibration process. This opens up the choices to a wide range of 2-port components, such as a transmission line, pair of matched loads, or an attenuator. This standard will determine 2 of the error terms.

For the final standard (C_3) only one piece of information is needed. This could be an unknown reflection value for the same reflection connected to each port ($S_{11} = S_{22}$). Since the other standards have been well matched, this standard should have a higher reflection. This standard determines the last error term.

The table shows a partial list of possible calibration configurations with appropriate three letter acronyms.

Example: TRL

Now to determine A, given X is known.

From $M = X\,A\,Y$, solve for A.

$$A = X^{-1} M\, Y^{-1}$$

From $M_1 = X C_1 Y$, solve for Y^{-1}.

$Y^{-1} = M_1^{-1} X\, C_1$, then finally solve for A.

$$\boxed{A = X^{-1} M\, M_1^{-1} X\, C_1}$$

The unknown device characteristics can be easily calculated by knowing the parameters of the X error adapter, the known standard C_1, and the measured data of the test device and measured data for C_1.

Example: Unknown T, Known A & B

$$\mathbf{T_M} = \frac{1}{(e_{10}e_{32})}\begin{bmatrix} -\Delta_X & e_{00} \\ -e_{11} & 1 \end{bmatrix}\mathbf{T}\begin{bmatrix} -\Delta_Y & e_{22} \\ -e_{33} & 1 \end{bmatrix} = \frac{1}{(e_{10}e_{32})}\mathbf{ATB}$$

$$(e_{10}e_{32})\mathbf{T_M} = \mathbf{ATB}$$

$$\det\left[(e_{10}e_{32})\mathbf{T_M}\right] = \det[\mathbf{ATB}]$$

$$(e_{10}e_{32})^2\det\mathbf{T_M} = \det[\mathbf{AB}], \text{ since } \det\mathbf{T} = 1, \text{ because } S_{21} = S_{12}$$

Therefore

$$(e_{10}e_{32}) = \pm\sqrt{\frac{\det\mathbf{A}\det\mathbf{B}}{\det\mathbf{T_M}}}$$

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This example is for the unknown thru calibration method (UXYZ). This is most easily developed using the cascaded t-parameter formulation. With 3 known standards at port-1 and port-2, 6 conditions are provided. The thru standard with $S_{21} = S_{12}$ provides the 7th required condition. The key to solving this approach is that the determinant of T is unity for the passive thru calibration connection ($S_{21} = S_{12}$).

Example: Unknown B, Known A & T

$$\mathbf{T_M} = \frac{1}{(e_{10}e_{32})}\begin{bmatrix} -\Delta_X & e_{00} \\ -e_{11} & 1 \end{bmatrix}\mathbf{T}\begin{bmatrix} -\Delta_Y & e_{22} \\ -e_{33} & 1 \end{bmatrix} = \frac{1}{(e_{10}e_{32})}\mathbf{ATB}$$

$$\frac{1}{(e_{10}e_{32})}\mathbf{B} = \mathbf{T}^{-1}\mathbf{A}^{-1}\mathbf{T_M} = \mathbf{D}, \text{ and } \mathbf{D} \text{ is completely known}$$

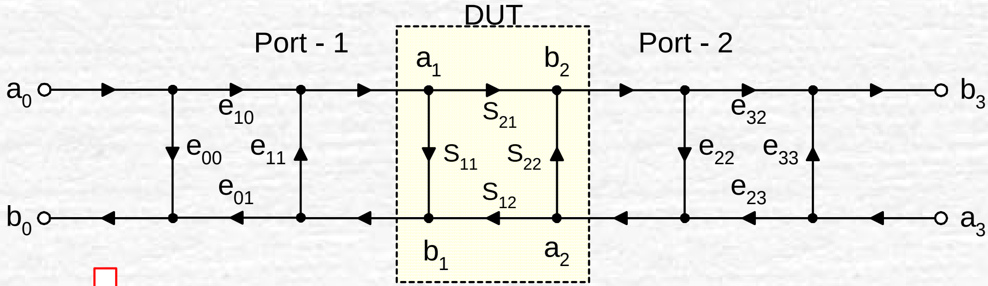
$$\begin{bmatrix} -\frac{\Delta_Y}{(e_{10}e_{32})} & \frac{e_{22}}{(e_{10}e_{32})} \\ -\frac{e_{33}}{(e_{10}e_{32})} & \frac{1}{(e_{10}e_{32})} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}$$

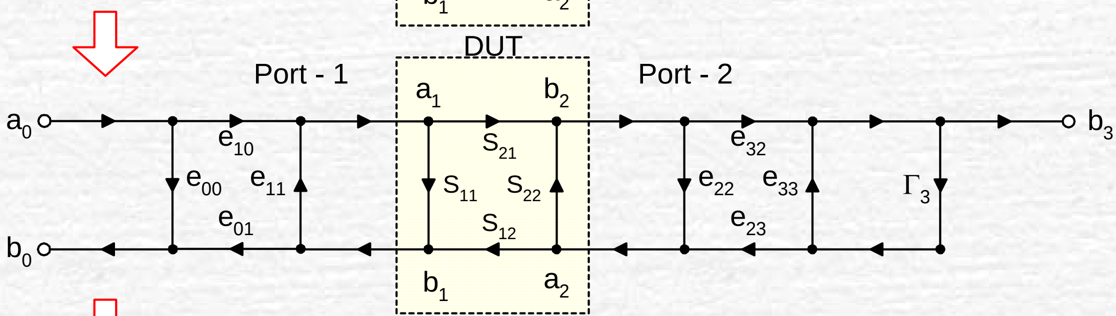
Therefore

$$(e_{10}e_{32}) = \frac{1}{D_{22}} \qquad e_{22} = \frac{D_{12}}{D_{22}} \qquad e_{33} = -\frac{D_{21}}{D_{22}} \qquad \Delta_Y = -\frac{D_{11}}{D_{22}}$$

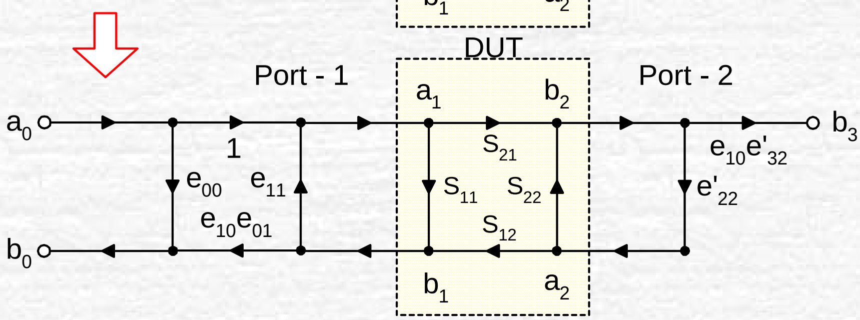
This example calibration approach (TXYZ) requires 3 known standards on port-1 but none on port-2 as long as the thru connection is known. The known thru connection provides 4 observations. The 3 known reflection standards connected to port-1 complete the required 7 known conditions.

8-Term to 10-Term -Forward





$$\Gamma_3 = \frac{a_3}{b_3}$$

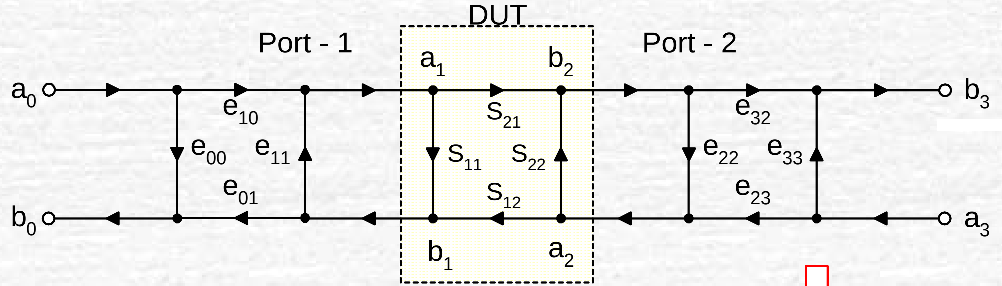


$$e'_{22} = e_{22} + \frac{e_{32}e_{23}\Gamma_3}{1-e_{33}\Gamma_3}$$

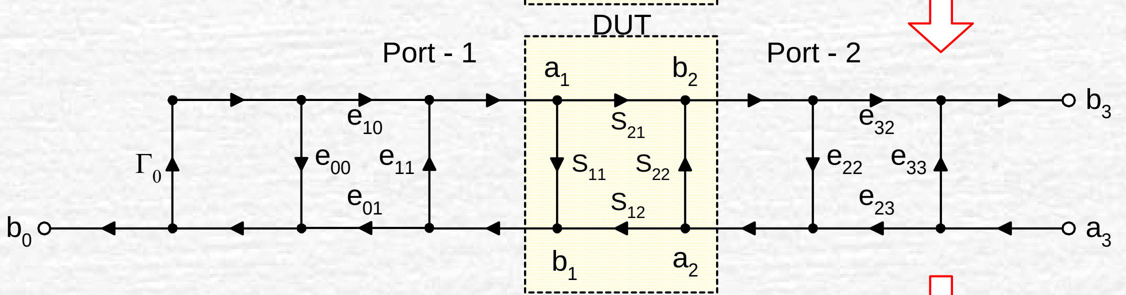
$$e'_{32} = \frac{e_{32}}{1-e_{33}\Gamma_3}$$

The 12-term and 8-term models describe the same system. So there must be a relationship between them. First lets reduce the 12 term model to 10 terms by removing the crosstalk terms which can easily be measured in a separate step. Then the 8-term model can be modified as shown above. First the 4th measurement channel (a_3) is removed by defining the 'switch match' as Γ_3. Then the forward model error terms for port match and transmission tracking (e'_{22} and e'_{32}) can be calculated. This gives the standard forward model if we form the two products $e_{10}e_{01}$ and $e_{10}e'_{32}$ and normalize e_{10} to 1.

8-Term to 10-Term -Reverse

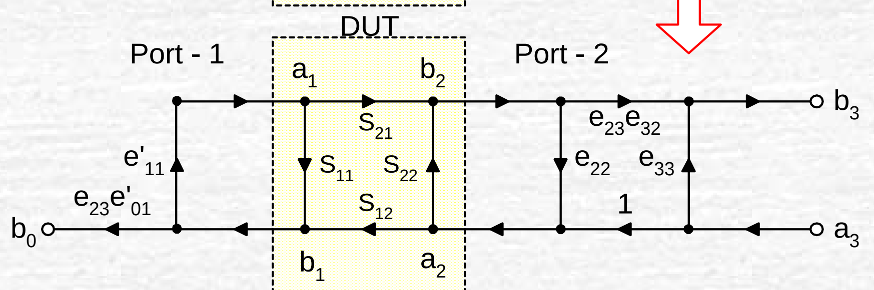


$$\Gamma_0 = \frac{a_0}{b_0}$$



$$e'_{11} = e_{11} + \frac{e_{10}e_{01}\Gamma_0}{1 - e_{00}\Gamma_0}$$

$$e'_{01} = \frac{e_{01}}{1 - e_{00}\Gamma_0}$$



The same procedure can be used for the reverse model.

There is a constraining relationship for the 8-term and 10-term models. This is the same as saying that the 8-term model can reduce to 7 terms when making ratio measurements. And the 10-term model can reduce to 9 terms.

8-term: $(e_{10}e_{01})(e_{23}e_{32}) = (e_{10}e_{32})(e_{23}e_{01})$

10-term: $[e_{10}e_{01} + e_{00}(e'_{11} - e_{11})][e_{23}e_{32} + e_{33}(e'_{22} - e_{22})] = (e_{10}e'_{32})(e_{23}e'_{01})$

.

Vector Network Analyzer References

Basic Error Correction Theory

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