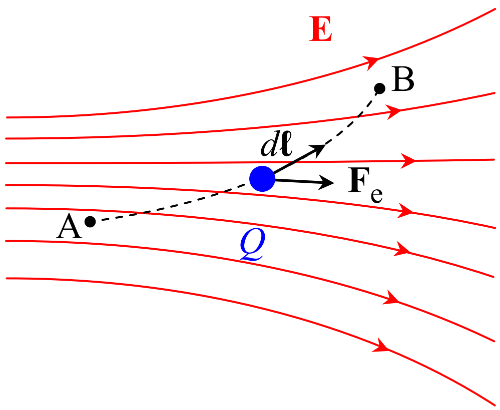
# 6 Eddy Current Inspection

# 6.1 Fundamentals

# Electric Field and Potential

$$\mathbf{F}_e = Q\,\mathbf{E}$$




$$dW = -\mathbf{F}_e \bullet d\boldsymbol{\ell}$$

$$W_{AB} = -Q\int_A^B \mathbf{E}\bullet d\boldsymbol{\ell}$$

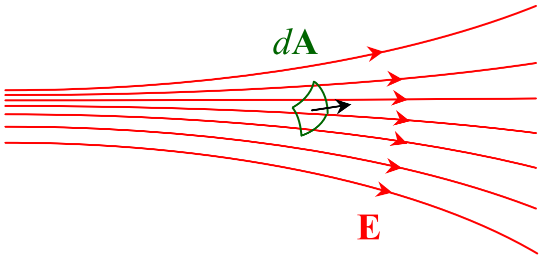
$$\Delta U = U_B - U_A = W_{AB}$$

$$U = V\,Q$$

$$\Delta V = V_B - V_A = -\int_A^B \mathbf{E}\bullet d\boldsymbol{\ell}$$

| | |
|---|---|
| $W$ | work done by moving the charge |
| $\mathbf{F}_e$ | Coulomb force |
| $\boldsymbol{\ell}$ | path length |
| $\mathbf{E}$ | electric field |
| $Q$ | charge |
| $U$ | electric potential energy of the charge |
| $V$ | potential of the electric field |

# Current, Current Density, and Conductivity




$$I = \frac{dQ}{dt}$$

$$I = \int \mathbf{J} \cdot d\mathbf{A}$$

$$dI = \mathbf{J} \cdot d\mathbf{A}$$

$$dQ = -ne\mathbf{v}_\mathrm{d} \cdot d\mathbf{A}\, dt$$

$$\mathbf{J} = -ne\mathbf{v}_\mathrm{d}$$

$$\frac{\mathbf{v}_\mathrm{d}}{\tau} = -\mathbf{E}\frac{e}{m}$$

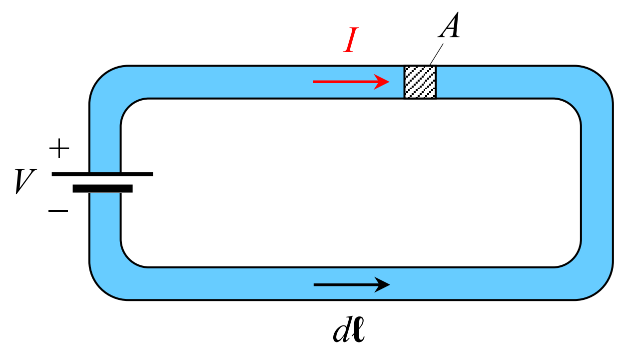
$$\tau = \frac{\Lambda}{v}$$

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$\mathbf{J} = \frac{ne^2\tau}{m}\mathbf{E} = \sigma\mathbf{E}$$

| | |
|---|---|
| $I$ | current |
| $Q$ | transferred charge |
| $t$ | time |
| $\mathbf{J}$ | current density |
| $\mathbf{A}$ | cross section area |
| $n$ | number density of electrons |
| $\mathbf{v}_\mathrm{d}$ | mean drift velocity |
| $e$ | charge of proton |
| $m$ | mass of electron |
| $\tau$ | collision time |
| $\Lambda$ | free path |
| $v$ | thermal velocity |
| $k$ | Boltzmann's constant |
| $T$ | absolute temperature |
| $\sigma$ | conductivity |

# Resistivity, Resistance, and Ohm's Law




$$V = V_+ - V_- = -\int_{S_-}^{S_+} \mathbf{E} \bullet d\boldsymbol{\ell}$$

$$V = \int_0^L \frac{J}{\sigma} d\ell = I \int_0^L \frac{d\ell}{\sigma A}$$

$$R = \frac{V}{I}$$

$$P = \frac{dU}{dt} = V \frac{dQ}{dt} = V I$$

| | |
|---|---|
| $V$ | voltage |
| $I$ | current |
| $R$ | resistance |
| $P$ | power |
| $\sigma$ | conductivity |
| $\rho$ | resistivity |
| $L$ | length |
| $A$ | cross section area |

$$R = \int_0^L \frac{d\ell}{\sigma A} = \int_0^L \frac{\rho\, d\ell}{A}$$

$$\rho = \frac{1}{\sigma}$$

$$R = \sum \frac{\rho_i L_i}{A_i}$$

$$R = \frac{\rho L}{A}$$

# Maxwell's Equations

## Field Equations:

| | |
|---|---|
| Ampère's law: | $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$ |
| Faraday's law: | $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ |
| Gauss' law: | $\nabla \bullet \mathbf{D} = q$ |
| Gauss' law: | $\nabla \bullet \mathbf{B} = 0$ |

## Constitutive Equations:

| | |
|---|---|
| conductivity | $\mathbf{J} = \sigma \mathbf{E}$ |
| permittivity | $\mathbf{D} = \varepsilon \mathbf{E}$ |
| permeability | $\mathbf{B} = \mu \mathbf{H}$ |

$\varepsilon = \varepsilon_0 \varepsilon_r \qquad (\varepsilon_0 \approx 8.85 \times 10^{-12}$ As/Vm)

$\mu = \mu_0 \mu_r \qquad (\mu_0 \approx 4\pi \times 10^{-7}$ Vs/Am)

# Electromagnetic Wave Equation

**Maxwell's equations:**

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} = -i\omega\mu\mathbf{H}$$

$$\nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{\partial t} = (\sigma+i\omega\varepsilon)\mathbf{E}$$

$$\nabla\cdot\mathbf{E} = 0$$

$$\nabla\cdot\mathbf{H} = 0$$

$$\nabla\times(\nabla\times\mathbf{E}) = -i\omega\mu(\sigma+i\omega\varepsilon)\mathbf{E}$$

$$\nabla\times(\nabla\times\mathbf{H}) = -i\omega\mu(\sigma+i\omega\varepsilon)\mathbf{H}$$

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$

$$\nabla^2\mathbf{E} = i\omega\mu(\sigma+i\omega\varepsilon)\mathbf{E}$$

$$\nabla^2\mathbf{H} = i\omega\mu(\sigma+i\omega\varepsilon)\mathbf{H}$$

**Wave equations:**

$$(\nabla^2 + k^2)\mathbf{E} = \mathbf{0}$$

$$(\nabla^2 + k^2)\mathbf{H} = \mathbf{0}$$

$$k^2 = -i\omega\mu(\sigma+i\omega\varepsilon)$$

**Example plane wave solution:**

$$\mathbf{E} = E_y\mathbf{e}_y = E_0 e^{i(\omega t - kx)}\mathbf{e}_y$$

$$\mathbf{H} = H_z\mathbf{e}_z = H_0 e^{i(\omega t - kx)}\mathbf{e}_z$$

# Wave Propagation versus Diffusion

$$k^2 = -i\omega\mu(\sigma + i\omega\varepsilon)$$

$k$ wave number

**Propagating wave in free space:**

$$k = \frac{\omega}{c} \qquad \mathbf{E} = E_0 \, e^{i\omega(t - x/c)} \, \mathbf{e}_y$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad \mathbf{H} = H_0 \, e^{i\omega(t - x/c)} \, \mathbf{e}_z$$

$c$ wave speed

**Propagating wave in dielectrics:**

$$c_\mathrm{d} = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_\mathrm{r}}} \qquad n = \frac{c}{c_\mathrm{d}} = \sqrt{\varepsilon_\mathrm{r}}$$

$n$ refractive index

**Diffusive wave in conductors:**

$$k = \sqrt{-i\omega\mu\sigma} = \frac{1}{\delta} - \frac{i}{\delta} \qquad \mathbf{E} = E_0 \, e^{-x/\delta} \, e^{i(\omega t - x/\delta)} \, \mathbf{e}_y$$

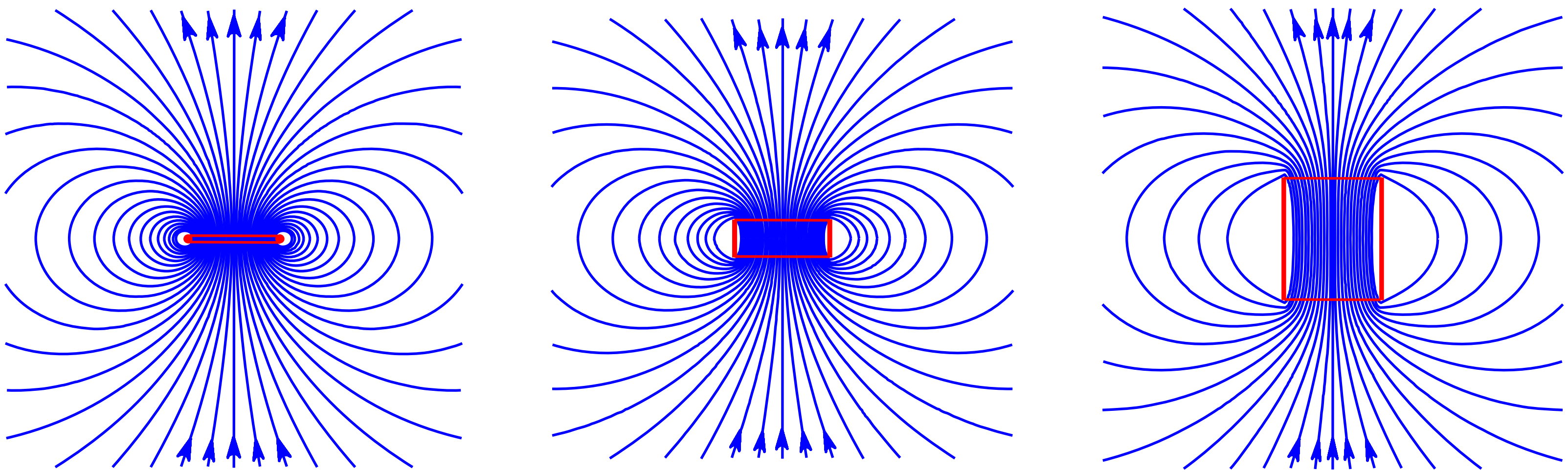
$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \qquad \mathbf{H} = H_0 \, e^{-x/\delta} \, e^{-i(\omega t - x/\delta)} \, \mathbf{e}_z$$

$\delta$ standard penetration depth

# 6.2 Eddy Currents

# Air-core Probe Coils

single turn $\qquad L = a \qquad L = 3\,a$



$$\mathbf{H} = \frac{I\,d\ell}{4\pi r^2}\,\mathbf{e}_\ell \times \mathbf{e}_r$$

$$H_{\text{center}} = \frac{I}{2a}$$

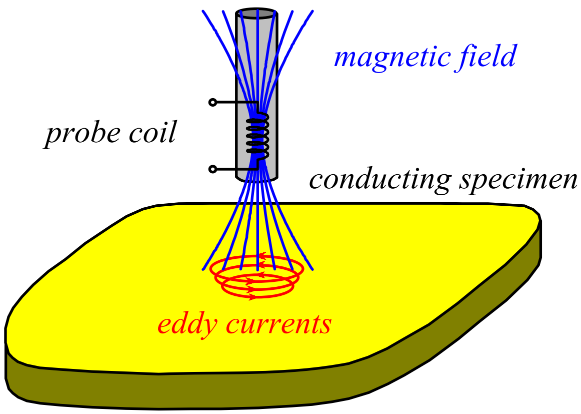
$L$ coil length

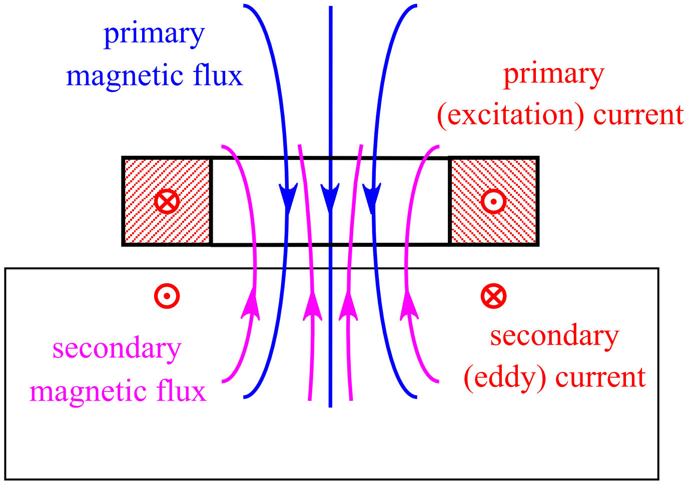
$a$ coil radius

$$\oint \mathbf{H} \bullet d\mathbf{s} = I_{\text{enc}}$$

$$\lim_{L/a \to \infty} H_{\text{center}} = \frac{N\,I}{L}$$

# Eddy Currents, Lenz's Law




$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

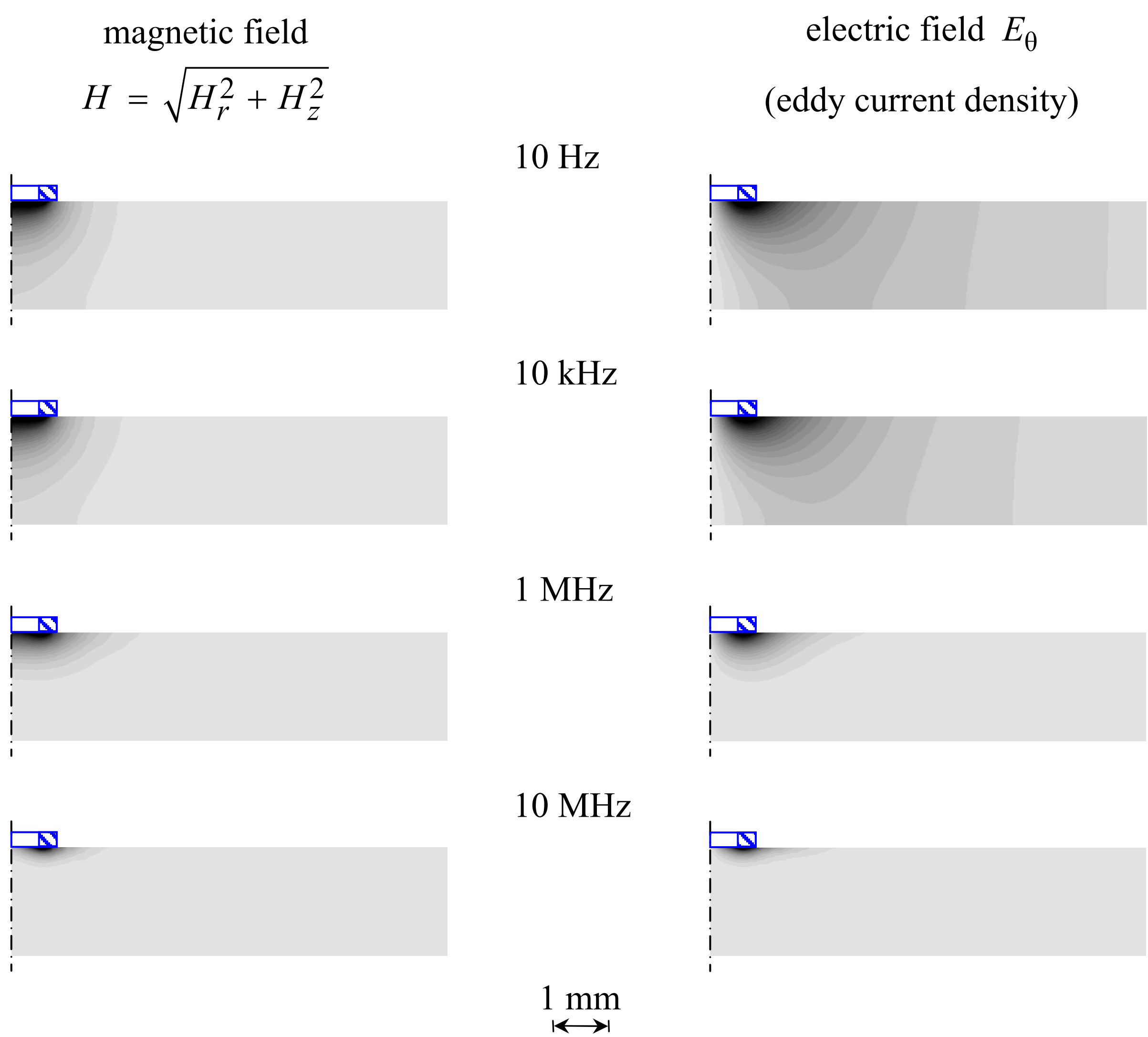
$$\nabla \times \mathbf{H}_\mathrm{p} = \mathbf{J}_\mathrm{p} \qquad \Phi_\mathrm{p} \propto \mu N I_\mathrm{p}$$

$$\nabla \times \mathbf{E}_\mathrm{s} = -\mu \frac{\partial}{\partial t}(\mathbf{H}_\mathrm{p} - \mathbf{H}_\mathrm{s}) \qquad V_\mathrm{s} = -\frac{d}{dt}(\Phi_\mathrm{p} - \Phi_\mathrm{s})$$

$$\mathbf{J}_\mathrm{s} = \sigma \mathbf{E}_\mathrm{s} \qquad I_\mathrm{s} \propto \sigma V_s$$

$$\nabla \times \mathbf{H}_\mathrm{s} = \mathbf{J}_\mathrm{s} \qquad \Phi_\mathrm{s} \propto \mu I_\mathrm{s} \Lambda_\mathrm{s}$$

# Field Distributions




air-core pancake coil ($a_i$ = 0.5 mm, $a_o$ = 0.75 mm, $h$ = 2 mm), in Ti-6Al-4V ($\sigma$ = 1 %IACS)

# Eddy Current Penetration Depth

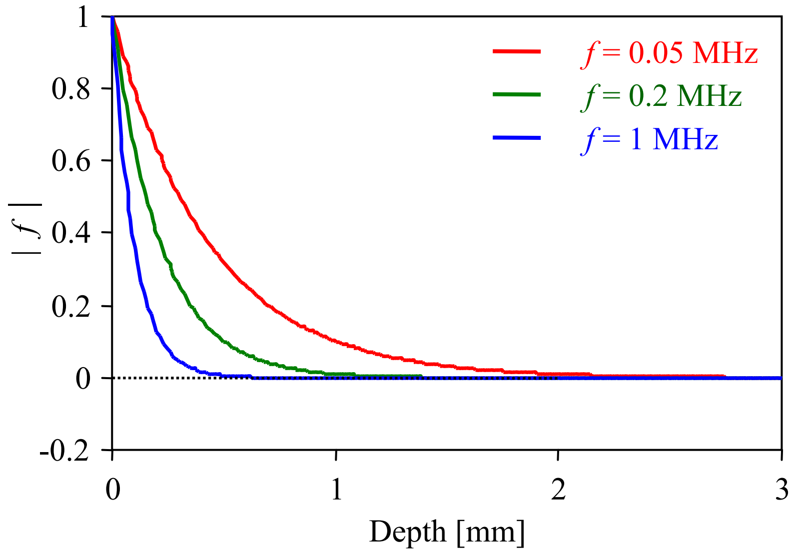
$$\mathbf{E} = E_0 \; f(x) \; e^{i\omega t} \; \mathbf{e}_y$$

$$\mathbf{H} = H_0 \; f(x) \; e^{i\omega t} \; \mathbf{e}_z$$

$$f(x) = e^{-x/\delta} \; e^{-i\,x/\delta}$$

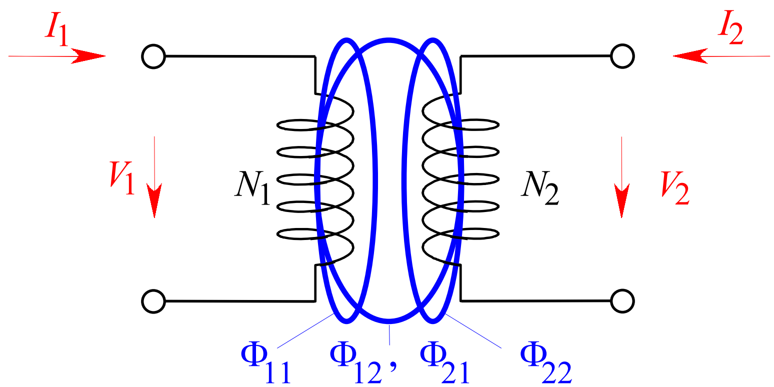
$\delta$ standard penetration depth

aluminum ($\sigma = 26.7 \times 10^6$ S/m or 46 %IACS)



# 6.3 Impedance Diagrams

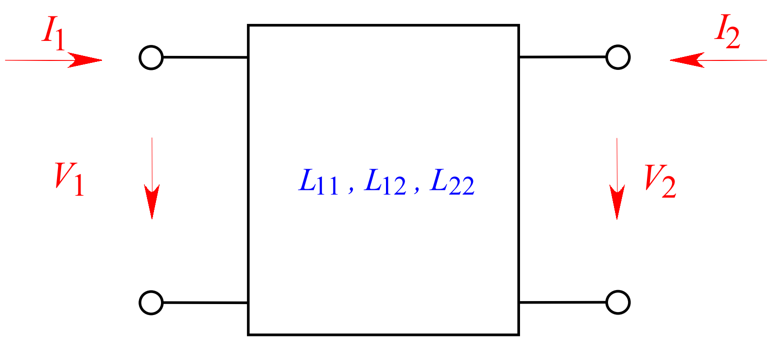
# Magnetic Coupling




$$\frac{\Phi_{12}}{\Phi_{22}} = \frac{\Phi_{21}}{\Phi_{11}} = \kappa$$

$$V_1 = N_1 \frac{d}{dt}(\Phi_{11} + \Phi_{12})$$

$$V_2 = N_2 \frac{d}{dt}(\Phi_{21} + \Phi_{22})$$




$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = i\omega \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

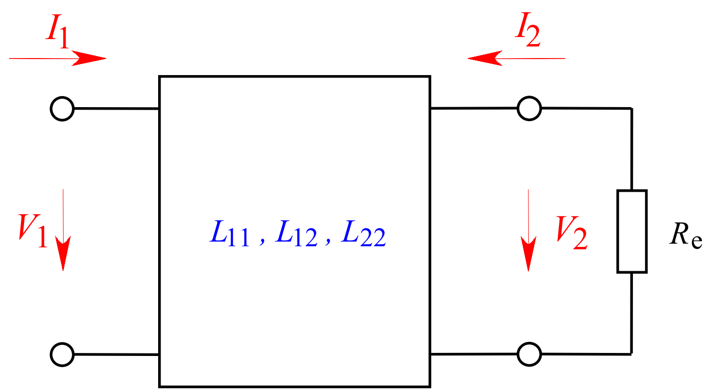
$$\Phi_{11} = \frac{I_1 L_{11}}{N_1} \qquad \Phi_{22} = \frac{I_2 L_{22}}{N_2}$$

$$\Phi_{21} = \kappa \Phi_{11} = \kappa \frac{I_1 L_{11}}{N_1} \qquad \Phi_{12} = \kappa \Phi_{22} = \kappa \frac{I_2 L_{22}}{N_2}$$

$$L_{21} = \kappa \frac{N_2}{N_1} L_{11} \qquad L_{12} = \kappa \frac{N_1}{N_2} L_{22}$$

$$L_{12} = L_{21} = \kappa \sqrt{L_{11} L_{22}}$$

# Probe Coil Impedance




$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = i\omega \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$L_{12}^2 = \kappa^2 L_{11} L_{22}$$

$$\kappa = \kappa(\ell)$$

$$\tilde{Z}_{\text{coil}} = \tilde{Z}_{\text{ref}} [1 + \xi(\omega, \sigma, \ell)]$$

$$\tilde{Z}_{\text{coil}} = \frac{V_1}{I_1}$$

$$\tilde{Z}_{\text{ref}} \approx i\omega L_{11}$$

$$\tilde{Z}_{\text{n}} = \frac{\tilde{Z}_{\text{coil}}}{\omega L_{11}} = i(1 + \xi)$$

$$V_2 = -I_2 R_{\text{e}} = i\omega L_{12} I_1 + i\omega L_{22} I_2$$

$$I_2 = \frac{-i\omega L_{12}}{R_{\text{e}} + i\omega L_{22}} I_1$$

$$V_1 = i\omega L_{11} I_1 + i\omega L_{12} I_2$$

$$V_1 = (i\omega L_{11} + \frac{\omega^2 L_{12}^2}{R_{\text{e}} + i\omega L_{22}}) I_1$$

$$\tilde{Z}_{\text{coil}} = i\omega L_{11} + \frac{\omega^2 L_{12}^2}{R_{\text{e}} + i\omega L_{22}}$$

$$\tilde{Z}_{\text{n}} = i + \kappa^2 \frac{\omega L_{22}}{R_e + i\omega L_{22}}$$

$$\tilde{Z}_{\text{n}} = i + \kappa^2 \frac{\omega L_{22}}{R_{\text{e}} + i\omega L_{22}} \frac{R_{\text{e}} - i\omega L_{22}}{R_{\text{e}} - i\omega L_{22}}$$

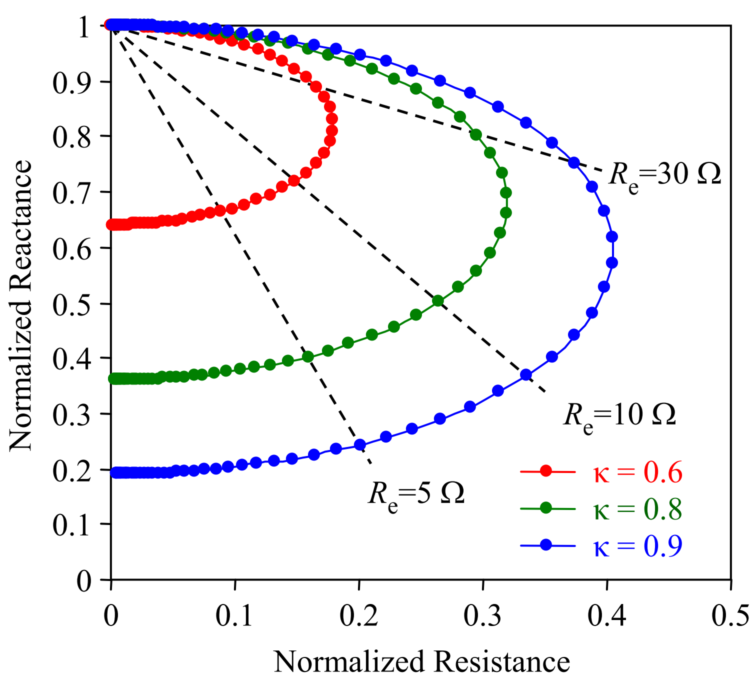
$$\tilde{Z}_{\text{n}} = \kappa^2 \frac{\omega L_{22} R_{\text{e}}}{R_{\text{e}}^2 + \omega^2 L_{22}^2} + i\,(1 - \kappa^2 \frac{\omega^2 L_{22}^2}{R_{\text{e}}^2 + \omega^2 L_{22}^2})$$

# Impedance Diagram

$$\zeta = \omega L_{22}/R_e$$

$$R_n = \mathrm{Re}\{\tilde{Z}_n\} = \kappa^2 \frac{\zeta}{1+\zeta^2} \qquad X_n = \mathrm{Im}\{\tilde{Z}_n\} = 1 - \kappa^2 \frac{\zeta^2}{1+\zeta^2}$$

$L_{22} = 3\ \mu\mathrm{H}, \quad f = 1\ \mathrm{MHz}, \quad \Delta R_e / R_e = 10\%$




lift-off trajectories are straight:

$$X_n = 1 - R_n \zeta$$

conductivity trajectories are semi-circles

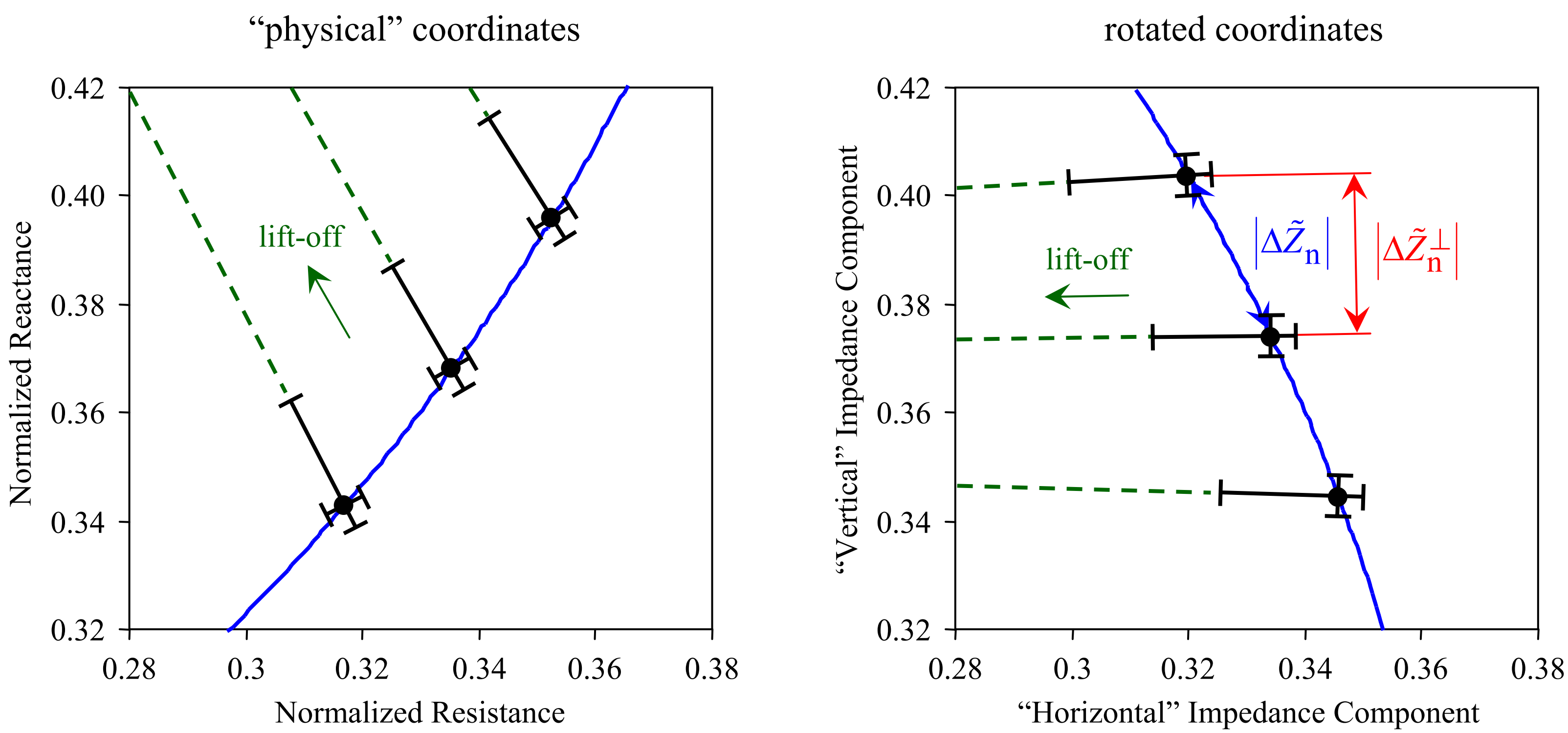
$$R_n^2 + \left( X_n - 1 + \frac{\kappa^2}{2} \right)^2 = \left( \frac{\kappa^2}{2} \right)^2$$

$$\lim_{\omega \to 0} R_n = 0 \quad \text{and} \quad \lim_{\omega \to 0} X_n = 1$$

$$\lim_{\omega \to \infty} R_n = 0 \quad \text{and} \quad \lim_{\omega \to \infty} X_n = 1 - \kappa^2$$

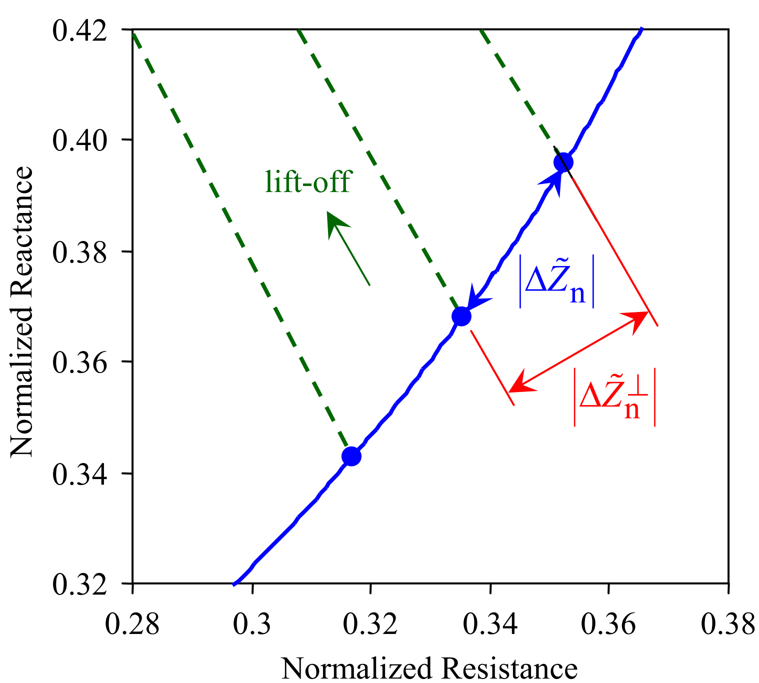
$$R_n(\zeta{=}1) = \frac{\kappa^2}{2} \quad \text{and} \quad X_n(\zeta{=}1) = 1 - \frac{\kappa^2}{2}$$

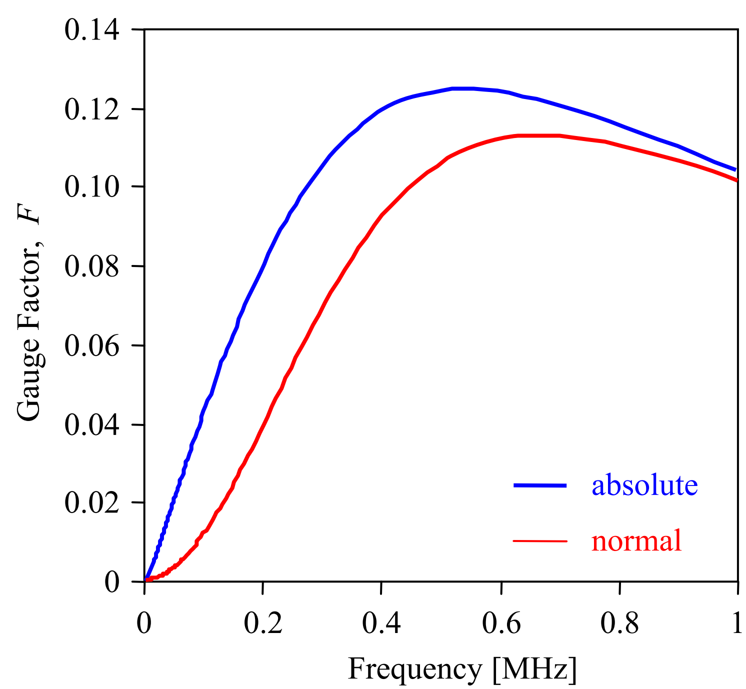
# Electric Noise versus Lift-off Variation



# Conductivity Sensitivity, Gauge Factor

$$L_{22} = 3\,\mu\text{H}, \quad f = 1\,\text{MHz}, \quad R_e = 10\Omega, \quad \Delta R_e = \pm 1\Omega$$

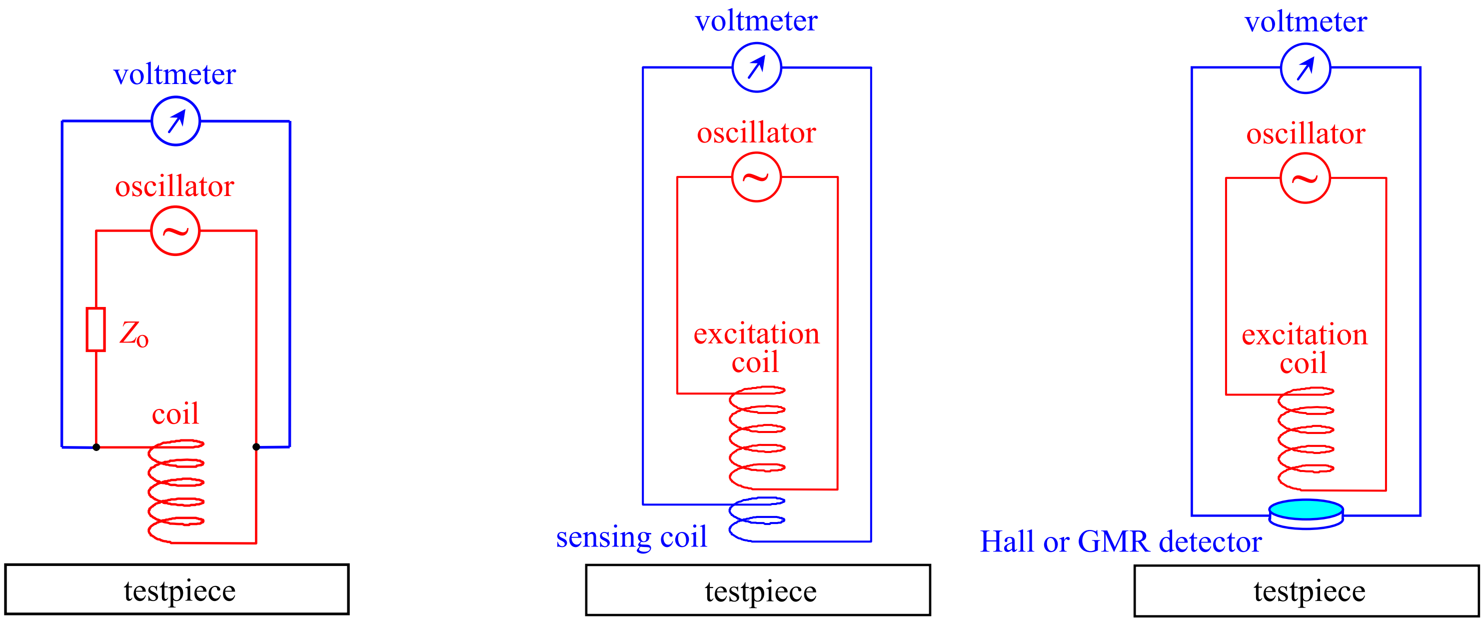


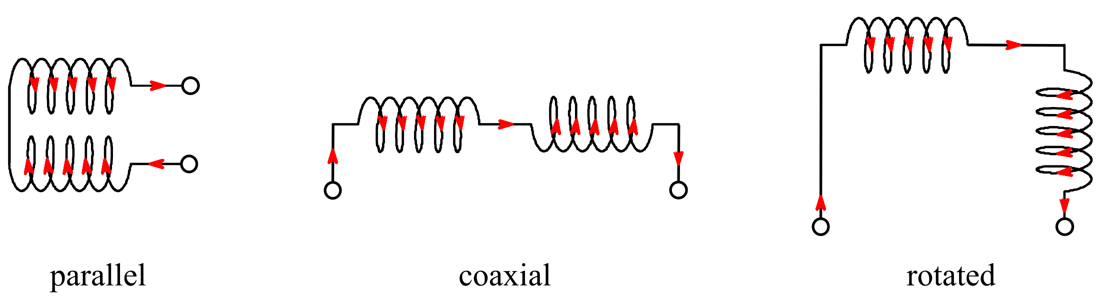

$$F_{abs} = \frac{|\Delta\tilde{Z}_n|}{\Delta R_e / R_e} \qquad F_{norm} = \frac{\Delta\tilde{Z}_n^{\perp}}{\Delta R_e / R_e}$$

# 6.4 Inspection Techniques

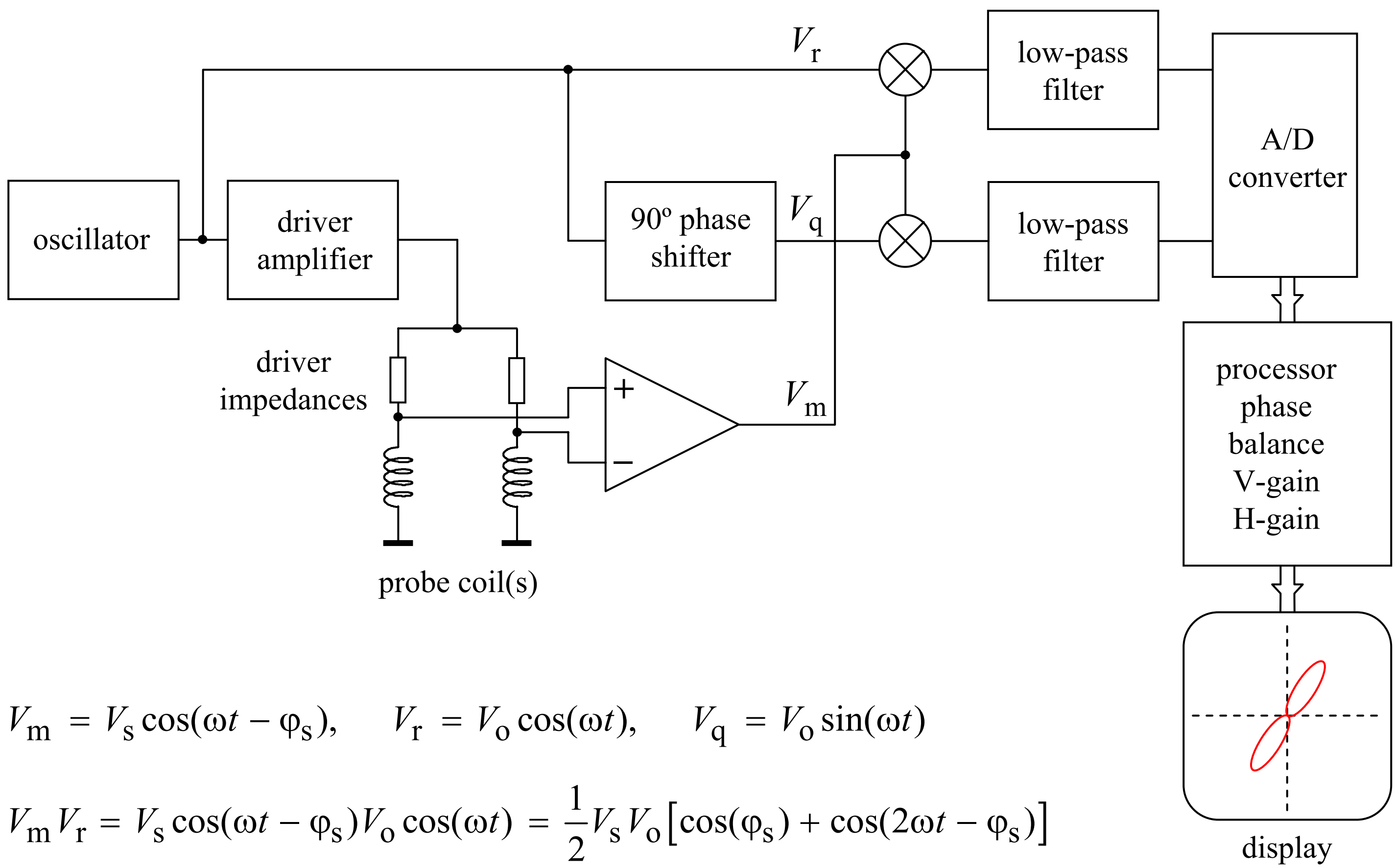
# Coil Configurations




differential coils



# Single-Frequency Operation




$$V_m = V_s\cos(\omega t - \varphi_s), \qquad V_r = V_o\cos(\omega t), \qquad V_q = V_o\sin(\omega t)$$

$$V_m V_r = V_s\cos(\omega t - \varphi_s)V_o\cos(\omega t) = \frac{1}{2}V_s V_o\left[\cos(\varphi_s) + \cos(2\omega t - \varphi_s)\right]$$

$$V_m V_q = V_s\cos(\omega t - \varphi_s)V_o\sin(\omega t) = \frac{1}{2}V_s V_o\left[\sin(\varphi_s) + \sin(2\omega t - \varphi_s)\right]$$

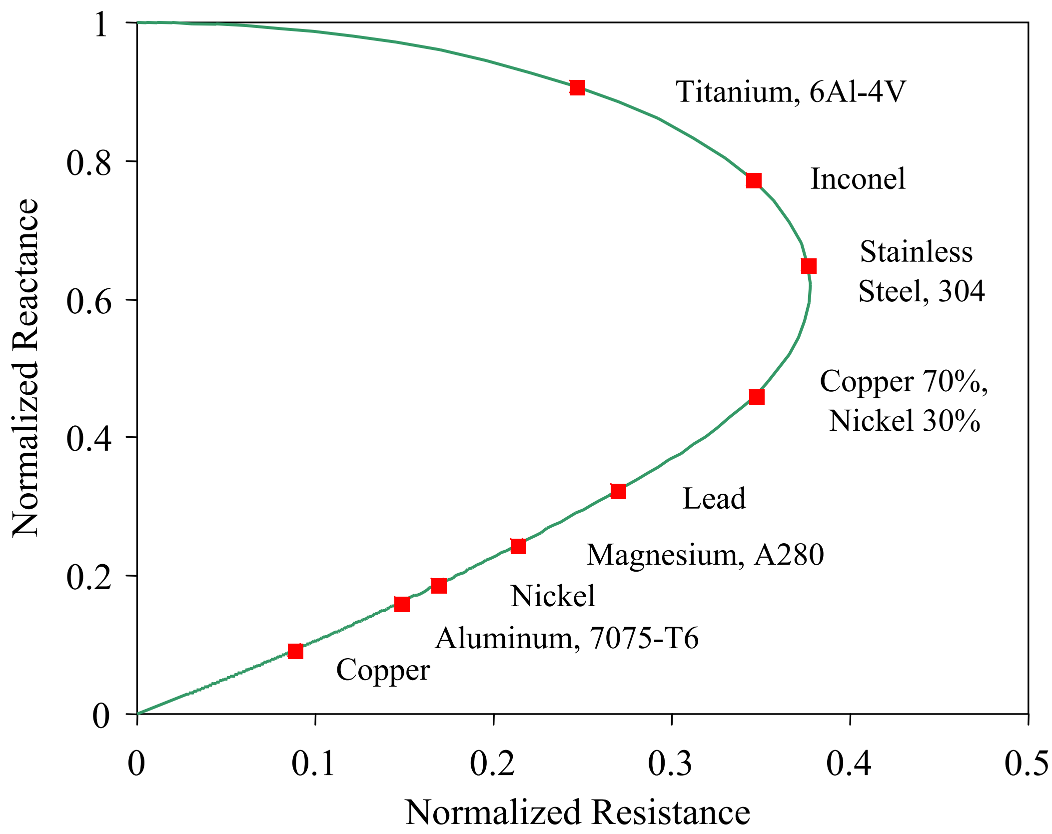
$$\overline{V_m V_r} = \frac{V_o}{2}V_s\cos(\varphi_s), \qquad \overline{V_m V_q} = \frac{V_o}{2}V_s\sin(\varphi_s)$$

# 6.5 Applications

- conductivity measurement
- permeability measurement
- metal thickness measurement
- coating thickness measurements
- flaw detection

# Conductivity versus Probe Impedance

constant frequency

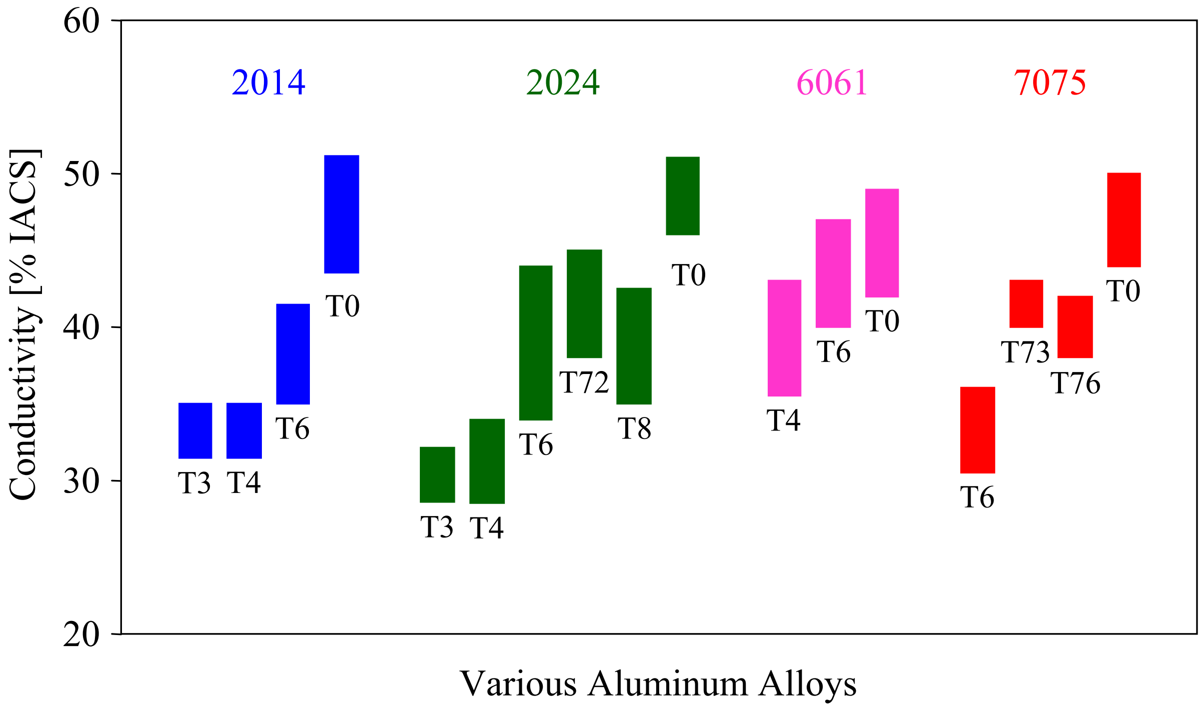


# Conductivity versus Alloying and Temper

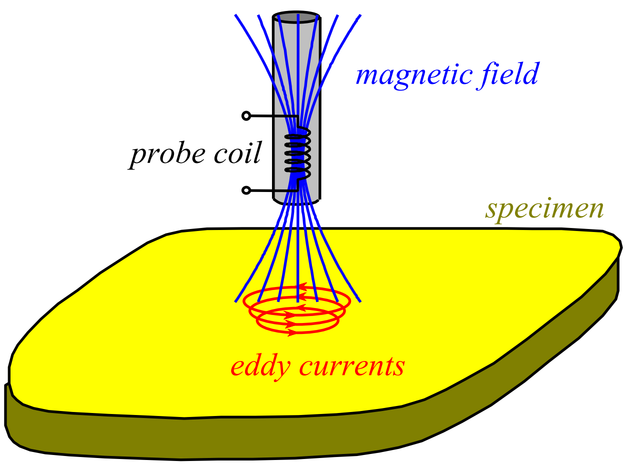
IACS = International Annealed Copper Standard

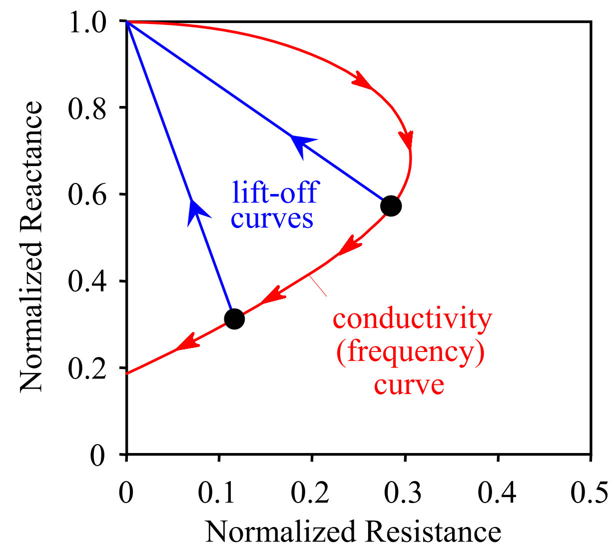
$\sigma_{IACS} = 5.8\times10^{7}\ \Omega^{-1}m^{-1}$ at 20 °C

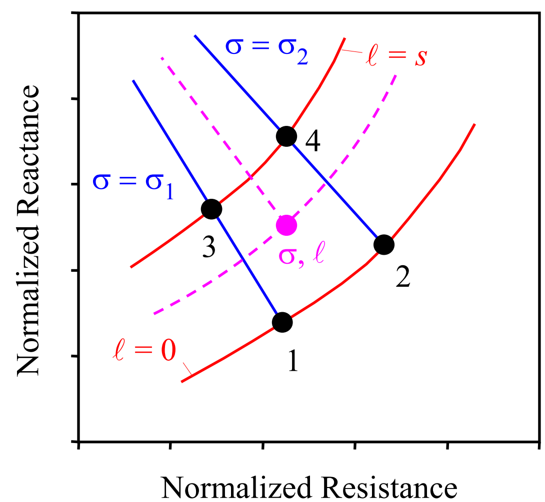
$\rho_{IACS} = 1.7241\times10^{-8}\ \Omega m$



# Apparent Eddy Current Conductivity



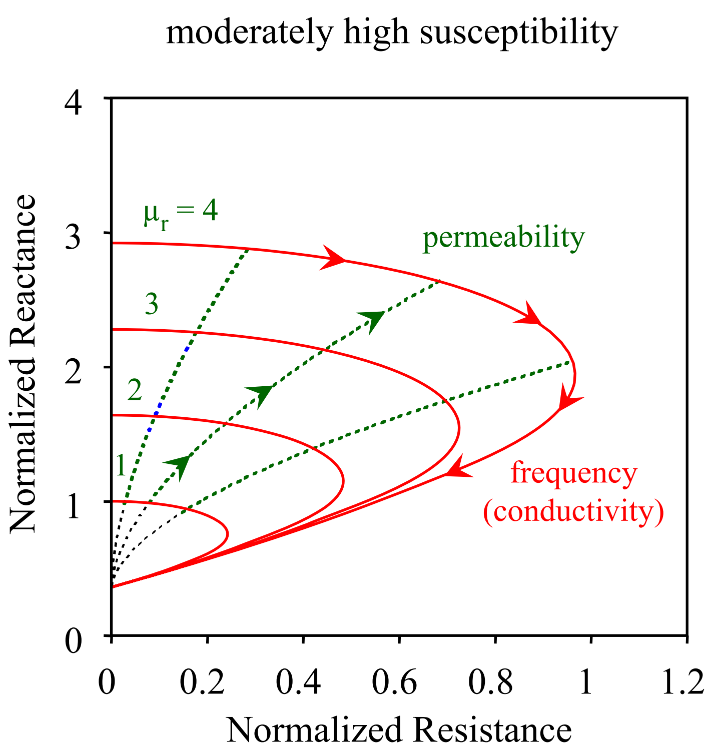



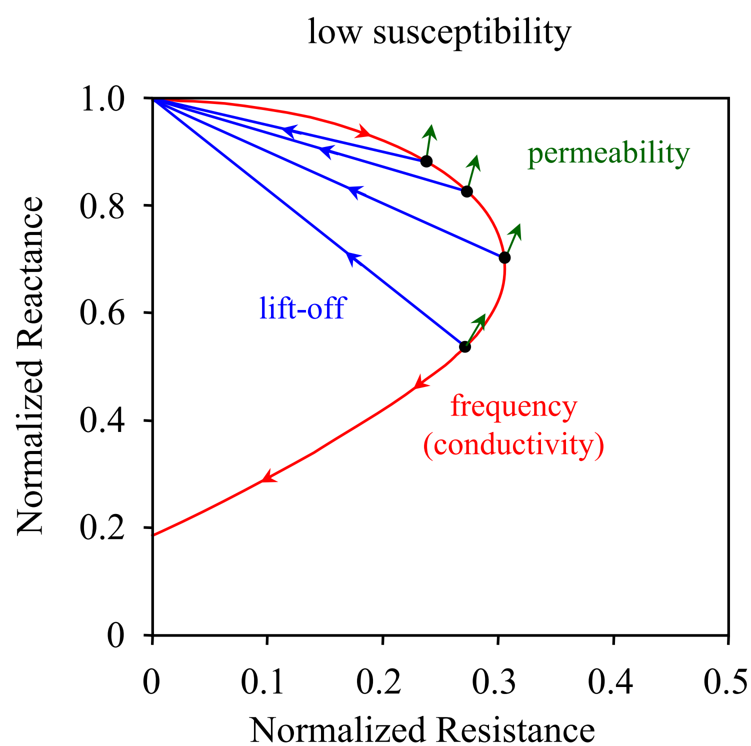




- high accuracy (≤ 0.1 %)
- controlled penetration depth

# Magnetic Susceptibility

paramagnetic materials with small ferromagnetic phase content

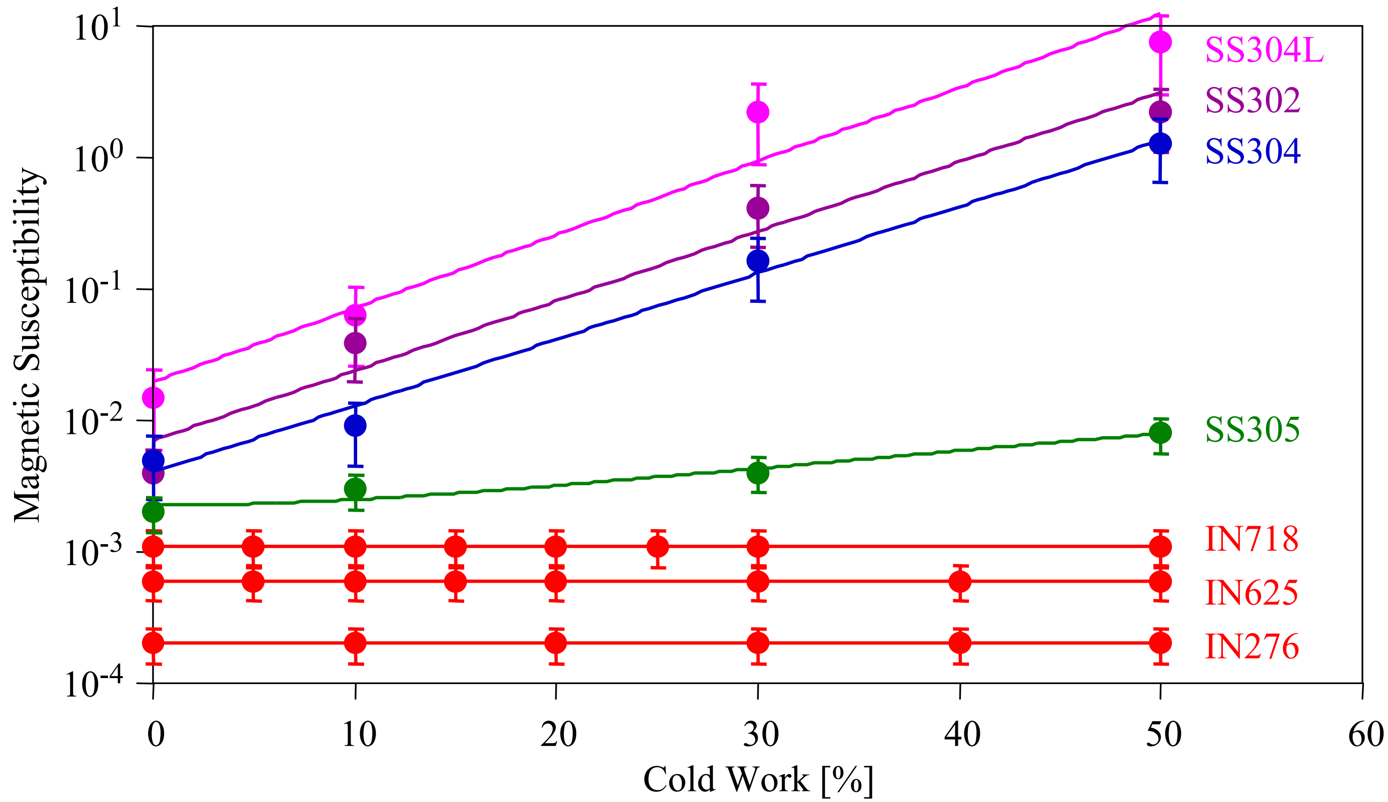


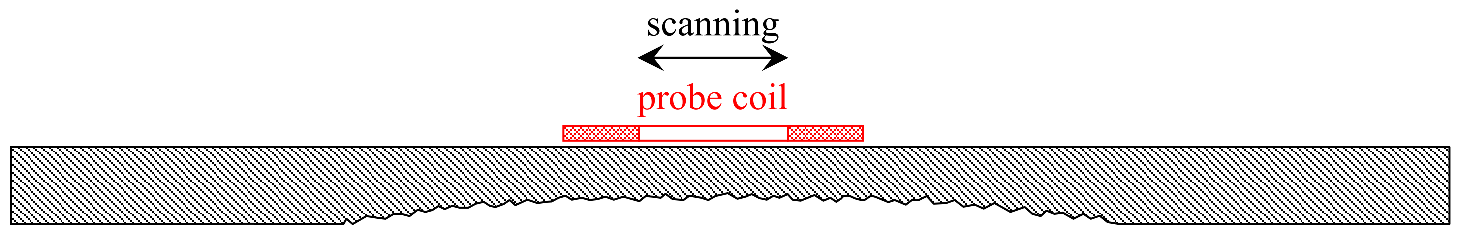

increasing magnetic susceptibility decreases the
apparent eddy current conductivity (AECC)

# Magnetic Susceptibility versus Cold Work

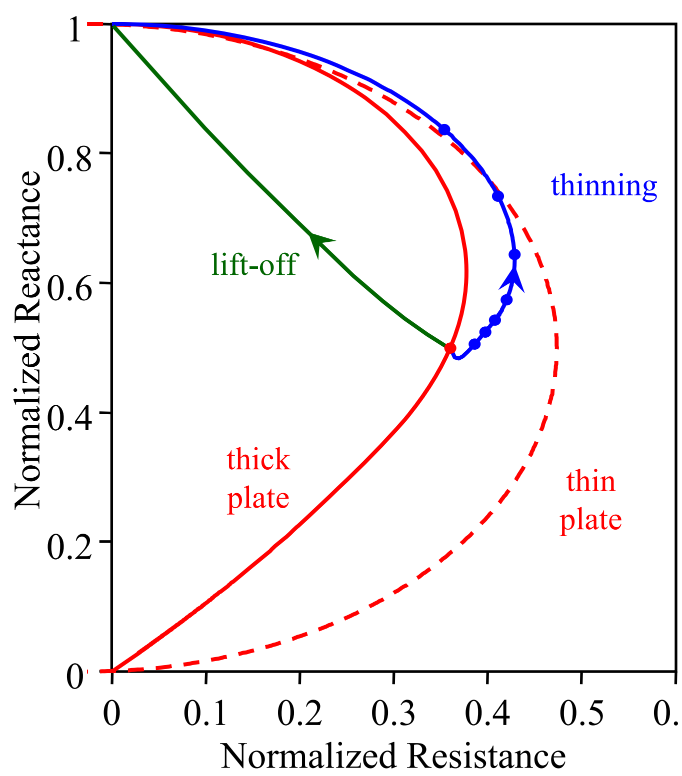
cold work (plastic deformation at room temperature) causes
martensitic (ferromagnetic) phase transformation
in austenitic stainless steels

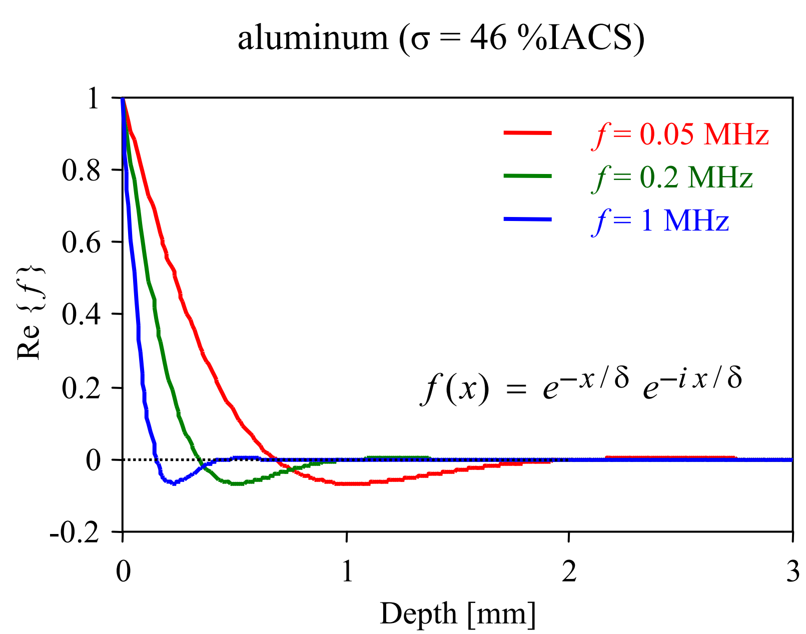


# Thickness versus Normalized Impedance

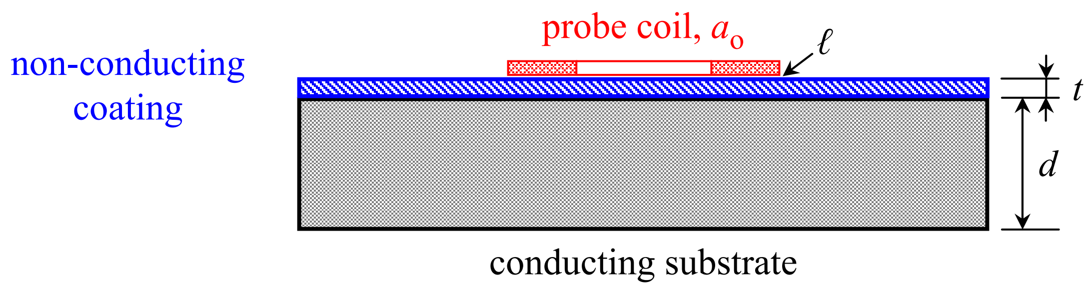


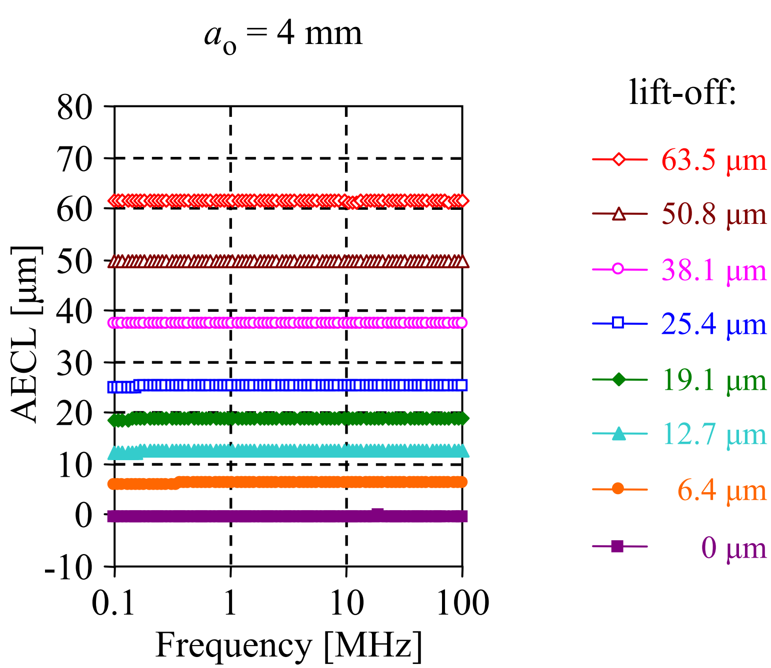

thickness loss due to corrosion, erosion, etc.



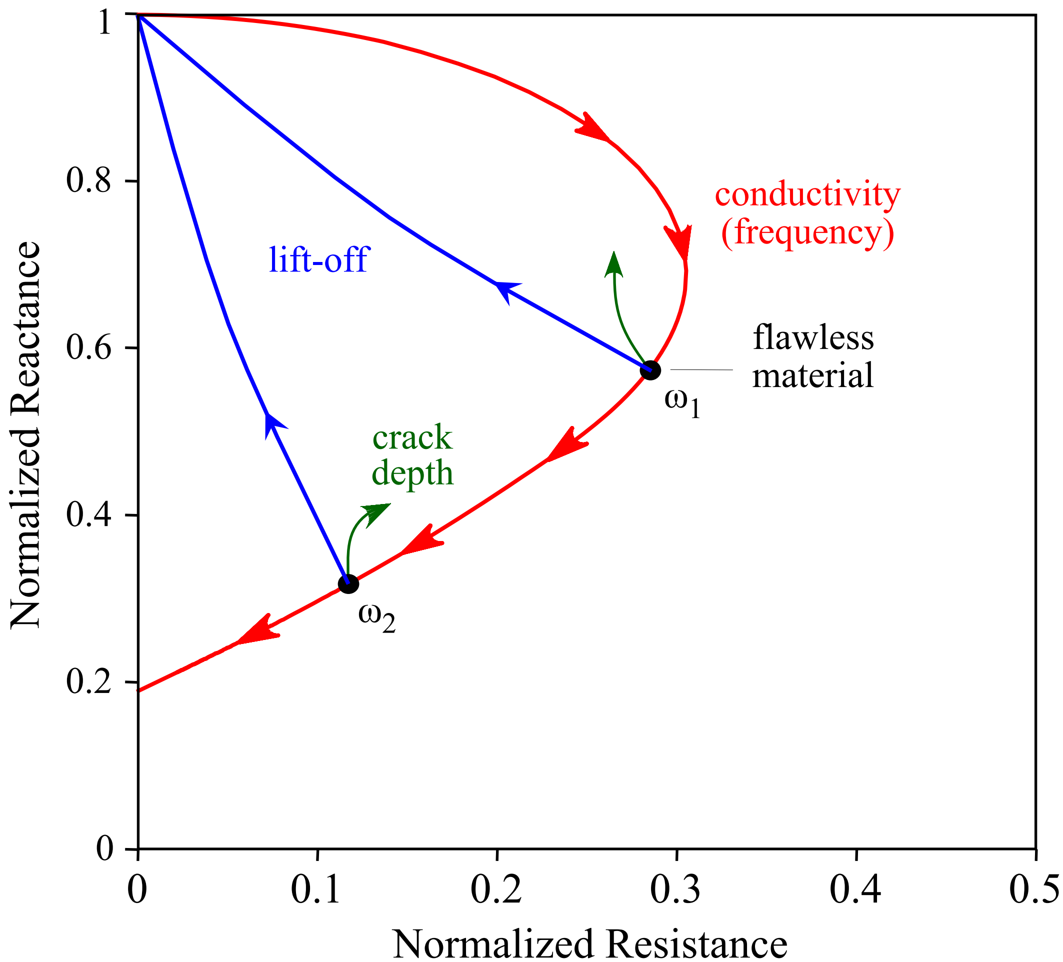



# Non-conducting Coating



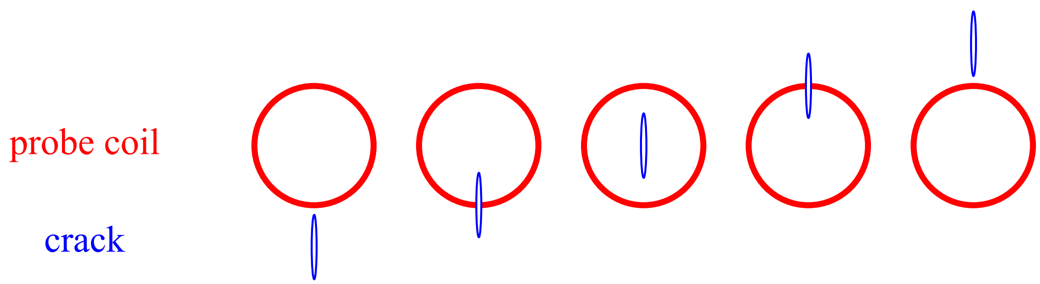



# Impedance Diagram



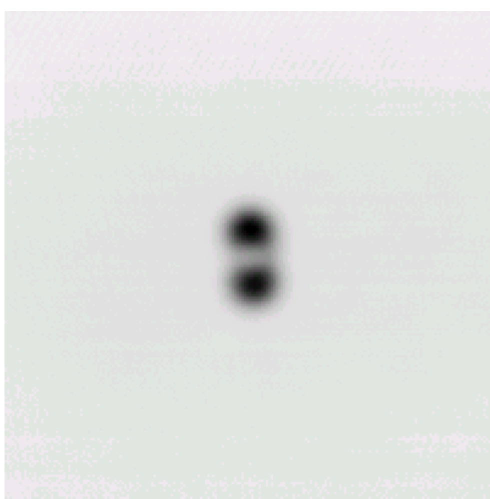

apparent eddy current conductivity (AECC) decreases
apparent eddy current lift-off (AECL) increases

# Eddy Current Images of Small Fatigue Cracks

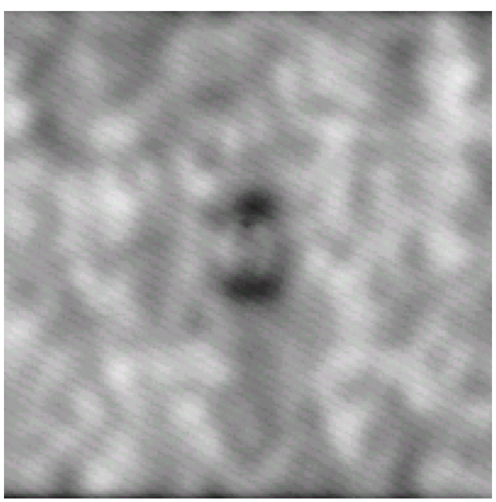



0.5” × 0.5”, 2 MHz, 0.060”-diameter coil

Al2024, 0.025” crack



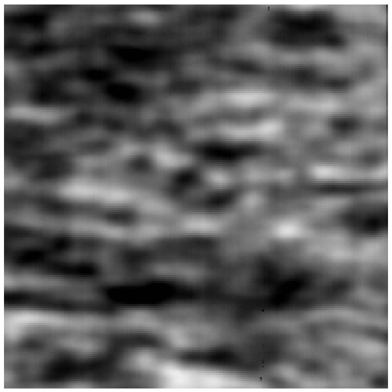
Ti-6Al-4V, 0.026”-crack



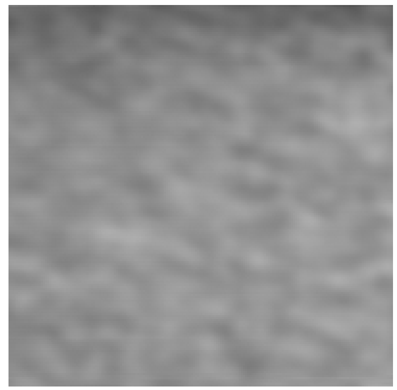
# Grain Noise in Ti-6Al-4V

1” × 1”, 2 MHz, 0.060”-diameter coil

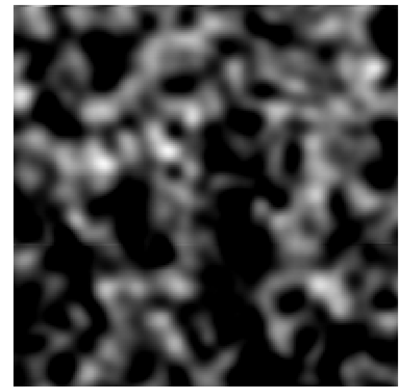
as-received billet material



solution treated and annealed

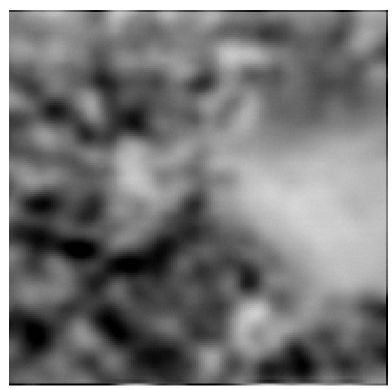


heat-treated, coarse

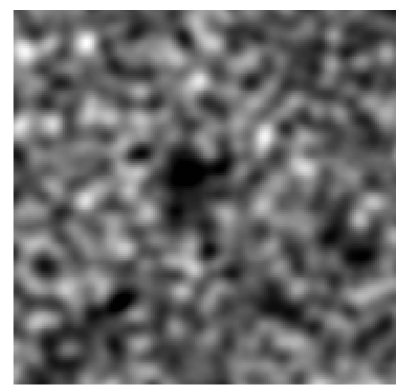


heat-treated, very coarse

heat-treated, large colonies



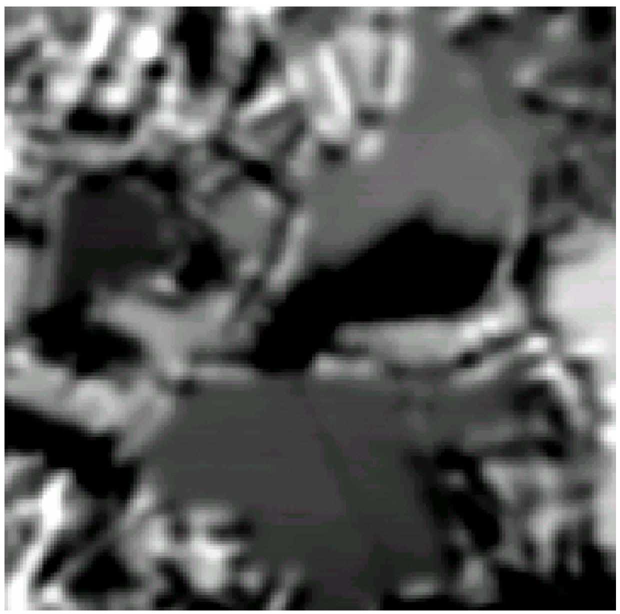
equiaxed beta annealed



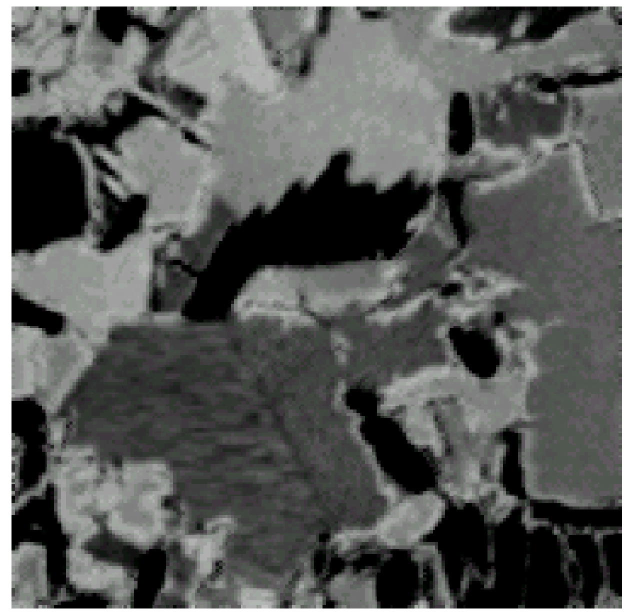
# Eddy Current versus Acoustic Microscopy

1” × 1”, coarse grained Ti-6Al-4V sample

5 MHz eddy current



40 MHz acoustic



# Inhomogeneity

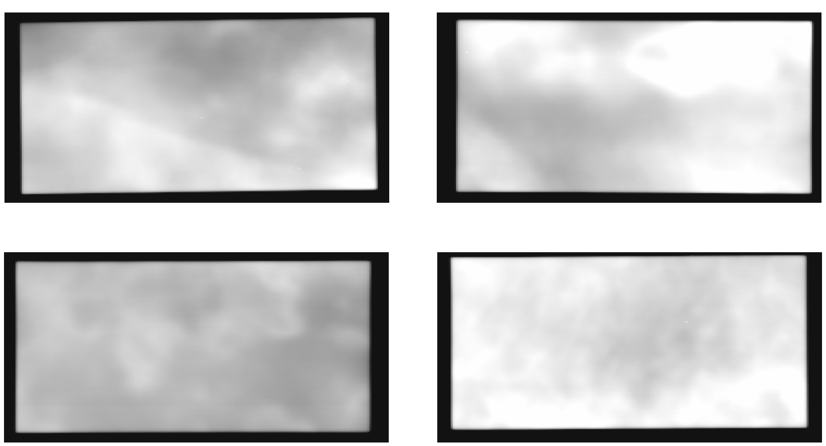
AECC Images of Waspaloy and IN100 Specimens

inhomogeneous Waspaloy

4.2” × 2.1”, 6 MHz

conductivity range ≈1.38-1.47 %IACS

±3 % relative variation



homogeneous IN100

2.2” × 1.1”, 6 MHz

conductivity range ≈1.33-1.34 %IACS

±0.4 % relative variation