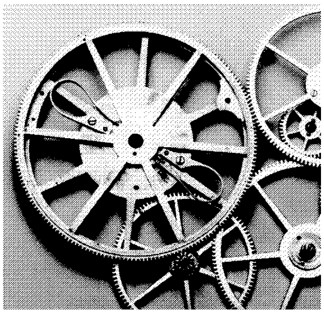
SOME PRACTICAL ASPECTS OF CYCLOIDAL GEARING



INTRODUCTION

The commonly available texts are very confusing when it comes to the practicalities of tooth dimensions for the wheels and pinions of clocks. For that matter, they're equally confusing if not evasive about the theory too[1]. This is a note about so much of that as it is necessary to know when setting out to cut a wheel or pinion.

For those whose memories have not retained the nomenclature of spur gear teeth, I trust the diagram at the end of the note will be sufficient.

Clocks use cycloidal gears. That is, the wheels and pinions have a tooth shape that is close to the cycloidal shape that gives a constant ratio of wheel speed to pinion speed as the teeth continually engage and disengage.[2]

Conventionally, the flanks of the teeth of wheels and pinions *inside* the pitch-circle-diameter are radial straight lines. Note that these line's being radial notionally entails using a different cutter for every different number of teeth.

In gears which drive, these radial straight lines are at equi-angular spacing proportional to the number of teeth in the gear. In driven gears, the angular space between teeth is opened markedly and the teeth are correspondingly narrowed. This provides the clearance which contributes to ensuring that contact between mating teeth does not occur before the line of centres, thereby reducing friction. It is discernible as backlash.

That part is easy. There is general agreement on the extent of the widening of the inter tooth spacing on pinions, and practically speaking the cutter manufacturer and your dividing head will look after that for you.

When you set out to make a wheel or pinion you will know the thickness of the teeth and the number of teeth. The relative thickness of the teeth in clock work is expressed using the "module" method.

"Module" means millimetres of pitch circle diameter per tooth or algebraically :

$$M = PCD/N$$

Before cutting the gear, you will need to know the outside diameter for the blank and also the depth of cut.

The outside diameter of the blank is equal to the pitch-circle-diameter plus twice the addendum height

$$OD = PCD + 2a$$

The PCD is known from the module and the number of teeth, by definition

$$PCD = M \times N$$

and thus $OD = M N + 2a$

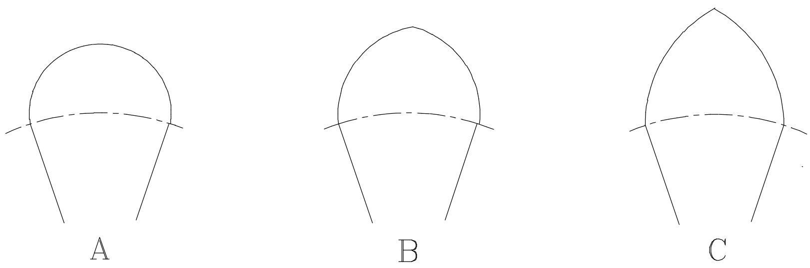
or as will prove useful later, $OD = M (N + 2a/M)$

The depth of cut is, of course, the sum of the addendum and dedendum heights. If the teeth are not to bottom in the spaces, the dedendum height must be greater than the addendum of the meshing gear.

What then is the height of the addendum? It is here the difficulty arises.

The size and shape of the addendum of driven gears, the pinions in clocks, are largely arbitrary. The addendum of the pinion is not supposed to be a working surface. In the first instance, pinions should work if they are cut off at the pitch circle and have no addenda. However, acceptable meshing in the face of manufacturing inaccuracies does have a bearing on the matter.

As a preliminary, note that the literature identifies three addendum shapes for pinions; one short and semi-circular, one a tall lancet shape, and the other an intermediate shape.



For driving gears, the wheels in a clock, the size and shape of the addendum are not arbitrary as the addendum is a working surface. The shape should be an epicycloid, but in practice a circular arc is used instead. The question facing horologists over the years has been how to choose a small standardised set of circular arcs to adequately approximate the infinite range of epicycloids required by the theory.

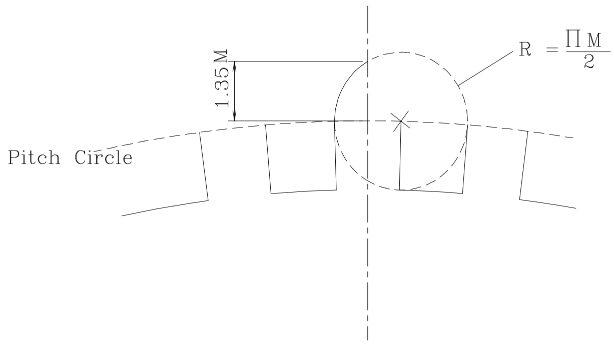
STANDARD DIMENSIONS

There are two common systems for standardising on the size and shape of the addendum of a driving wheel. Each of these systems has a corresponding method for sizing the dedendum and the mating pinion.

CONSTANT ADDENDUM SYSTEM

Driving Wheels

The first of these systems is the so called constant addendum or common addendum system. In this system the addendum of the wheel teeth is defined by a circular arc centred on the intersection of the pitch circle and the radial dedendum line of the opposite flank. The radius of the arc must equal the thickness of the tooth. The height of the addendum will be determined by the point where the addendum intersects the radial centre line of the tooth.



. This is known as the “full ogive” system.

The literature seems to be saying that wheel cutters made by Thorntons and Chronos followed this system,

Thorntons and Chronos, and presumably all other cutter manufacturers, choose to make their wheel tooth thickness equal to the inter-tooth space and therefore equal to half the circular pitch. Now the circular pitch of any gear is π M, So the thickness of the tooth is $\pi/2$ M

Accordingly, the radius of curvature of the addendum, which is equal to the tooth width [3], is equal to $\pi/2$ M.

ie: $R = \pi/2\ M$

In consequence, as shown in the diagram, *the height of the addendum comes out at 1.35M.* [4]

Thus the radius of the addendum arc and the addendum height are equal to the module multiplied by constants, numbers that that do not vary with the number of teeth in the wheel.

The height of the dedendum in this system seems to have been been fairly arbitrarily set also at $\pi/2$ M that is 1.57M - although you may see George Daniels, and Eliot Isaacs writing for Chronos, and others, write 1.55M. The difference is negligible and is, I believe the result of arithmetic rounding of the total tooth depth from 2.92M to 2.9M. In any case, this gives a very generous clearance for the tip of the pinion.

Thorntons use a wheel tooth dedendum of 2.0M for modules between 0.5 and 1.0.

For Wheels:
OD = (N + 2.7)M Depth of cut = 2.9M Depth of cut = 3.35M for Thornton 0.5<M>1.0

The dedendum flanks of conventional clock wheels are not actually radial as this would involve having a different cutter for each of a large set of possible teeth numbers. Instead a single cutter shape is used, for each module, giving flanks at an included angle of 3° corresponding to their being radial on a 60 tooth wheel.

Driven Pinions

The shape of pinion teeth to go with these wheels must be carefully specified, but it is not well recorded. The shape is defined in much the same way as for wheel teeth.

Tooth width is set first. The angular pitch of the teeth is apportioned between tooth and space. The tooth occupying 30% of the pitch for pinions having up to ten teeth, and 40% for pinions with eleven or more teeth.

The addendum radius of curvature is set by the choice of profile A, B, or C. Usually these apply to pinions of 10 or more teeth, 8 and 9 teeth, 6 and 7 teeth respectively - but not always. Any manufacturer is free to choose a profile radius of curvature, but standard radii have been specified.[5]

When applied to teeth of the two standard widths (30% and 40%) these standard profiles give a set of addendum heights and thus outside diameters. Remembering that the tooth width changes to 40% at eleven teeth, we obtain the following table.

Addendum Height for Pinions

Profile	6 to 10 teeth	11 teeth and over
A	0.525M	0.625M
B	0.670M	0.805M
C	0.855M	1.050M

This then leads to the pinion outside diameter, as follows.

Outside Diameter for Pinions [6]

Profile	6 to 10 teeth	11 teeth and over
A	(N + 1.05)M	(N + 1.25)M
B	(N + 1.34)M	(N + 1.61)M
C	(N + 1.71)M	(N + 2.10)M

The dedendum height of a pinion tooth is determined by the need to provide bottom clearance for the wheel tooth. The wheel tooth only engages to the depth of the addendum. Accordingly the pinion dedendum height need only be equal to the wheel addendum plus some clearance. There seems to be general agreement that this clearance should be 0.4M. This makes the pinion dedendum height 1.35M + 0.4M, that is 1.75M. Adding this to the pinion addendum we get the depth of cut, as follows.

Depth of Cut for Pinions

Profile	6 to 10 teeth	11 teeth and over
A	2.28M	2.38M
B	2.42M	2.56M
C	2.60M	2.80M

The dedendum of a pinion is cut radially and, in principle, this means a different cutter for every number of teeth.

It is up to the cutter manufacturer to decide which standard profile A B or C he will use for a given number of teeth. The cutter should be marked accordingly.

Sometimes compromises are used. Eliot Isaacs, writing for Chronos in 1979, implies that one manufacturer was then using a single tooth thickness of 1.15M, a single addendum profile having a radius of 0.8M and, in consequence, a single addendum height of 0.74M. (Leading to an OD of (N + 1.48)M and a depth of cut of 2.49M)

I infer from Malcom Wild that Thorntons do not use profile A. If that is the case the following apply to Thornton's full ogive cutters.

Outside Diameter Thornton Pinions ?

6,7,8 teeth	10 teeth	11 teeth and over
(N + 1.71)M	(N + 1.34)M	(N + 1.61)M

Depth of Cut for Thornton Pinions ?

6,7,8 teeth	10 teeth	11 teeth and over
2.60M	2.42M	2.56M

Very many clocks were built to this full ogive standard. constant addendum system. It is said to have been discarded in favour of a variable addendum system represented by BS 978 . In practice that is not correct as will be seen..

BRITISH STANDARD 978 PT2 SYSTEM

The second system for standardising on tooth profiles is set out in BS978 Pt2 and Swiss NHS 56702 and 56703. This is intended to give profiles representing the epicycloid rather more exactly than the constant addendum method. However, the principle is identical.

You will recall that in the constant addendum system the radius of curvature of the wheel addendum was given by $R = \pi/2$ M

Instead of a factor of $\pi/2$ the BS978 system standardises on a different "addendum radius factor" f_r for every wheel size from 18 teeth to 192 teeth. (The data is actually tabulated for pinion size and gear ratio.)

Further, instead of centering this radius and simply allowing the height to work itself out, in this system the addendum height is also specified for each wheel. This is done with an "addendum factor" *f*. The actual addendum height is *f* times the module M. [7] The centre of curvature is not specified directly in this method.

The wheel dedendum is still $\pi/2$ M in this system. Thus we get:

For Wheels:
OD = (N + 2*f*)M
Depth of cut = (1.57 + *f*)M

The pinions are defined in the way already described for the constant addendum system. The OD is as tabulated. However, since the pinion dedendum must match the wheel addendum the factor *f* is introduced, so that the pinion dedendum is (*f* + 0.4)M. Adding the variable addendum height of the pinion gives:

Depth of Cut for Pinions

Profile	6 to 10 teeth	11 teeth and over
A	(*f* + 0.925)M	(*f* + 1.025)M
B	(*f* + 1.070)M	(*f* + 1.205)M
C	(*f* + 1.255)M	(*f* + 1.450)M

THORNTON'S BS978 CONSTANT ADDENDUM SYSTEM

Such a system of cutter design is acceptable in a factory mass producing clocks or watches. The cutters are designed and made as part of the tooling for the production run. However in a jobbing shop the range of cutters required is prohibitively expensive. P.P. Thornton (Successors) Ltd markets a range of cutters for jobbing shops using a revised constant addendum system.

This system uses an addendum factor and corresponding addendum radius which BS 978 specifies for a wheel pinion pair with a wheel of about 45 teeth and a 7 tooth pinion. The included angle between the flanks of the wheel cutters is 4°, corresponding to a 45 tooth wheel.

The addendum factor *f* is 1.38. [8]

For the range of modules between 0.5 and 1.0 inclusive, Thornton introduces a deeper dedendum equal to 2M. So that :

OD = (N + 2.76)M
Depth of cut = 2.95M
Depth of cut = 3.38M for M = 0.5 to 1.0

These dimensions are probably negligibly different from the earlier "full ogive" constant addendum system which gave :

OD = (N + 2.7)M
Depth of cut = 2.9M
Depth of cut = 3.35M for M = 0.5 to 1.0

Thornton also make cutters for mainspring barrels where a short addendum is necessary.

Thornton pinion cutters follow BS978 with one minor exception..

The following tables summarise the current Thornton standard.

Wheels

Module M.	Up to and including 0.45, and 1.1 to 1.5	0.5 and up to and including 1.0	Short form 0.2 to 1.0
Pitch circle diameter	NxM	NxM	NxM
Tooth thickness	1.57 x M	1.57 x M	1.57 x M
Outside diameter	(N + 2.761 x M	(N + 2.761 x M	(N + 2.761 x M
Root diameter	(N-3.14)xM	(N - 4) x M	(N - 2.141 x M
Addendum radius	1.93 x M	1. 93 x M	1. 93 x M
Angle of cutter flank	2°	2°	2°
Addendum	1.38 x M	1.38 x M	1.38 x M
Dedendum	1.57 x M	2 x M	1.07 x M
Depth of Cut	2.95 x M	3.38 x M	2.45 x M

Pinions

Number of leaves	6	7	8	10	12	16
Pitch circle diameter	6M	7M	8M	10M	12M	16M
Tooth/pitch ratio	1/3	1/3	1/3	2/5	2/5	2/5
Leaf thickness	1.05M	1.05M	1.05M	1.25M	1.25M	1.25M
Outside diameter	7.71M	8.71M	9.71M	11.61M	13.61M	17.61M
Root diameter	2.5M	3.3M	4.2M	5.9M	4.8M	11.8M
Addendum radius	1.05M	1.05M	1.05M	0.82M	0.82M	0.82M
Angle of cutter flank	20°	1 7°-9'	15°	10°-48'	9°	6°-45'
Addendum	0.855M	0.855M	0.855MM	0.805M	0.805M	0.805M
Dedendum	1.75M	1.85M	1.90M	2.05M	2.10M	2.10M
Depth of Cut	2.605M	2.705M	2.755M	2.855M	2.905M	2.905M

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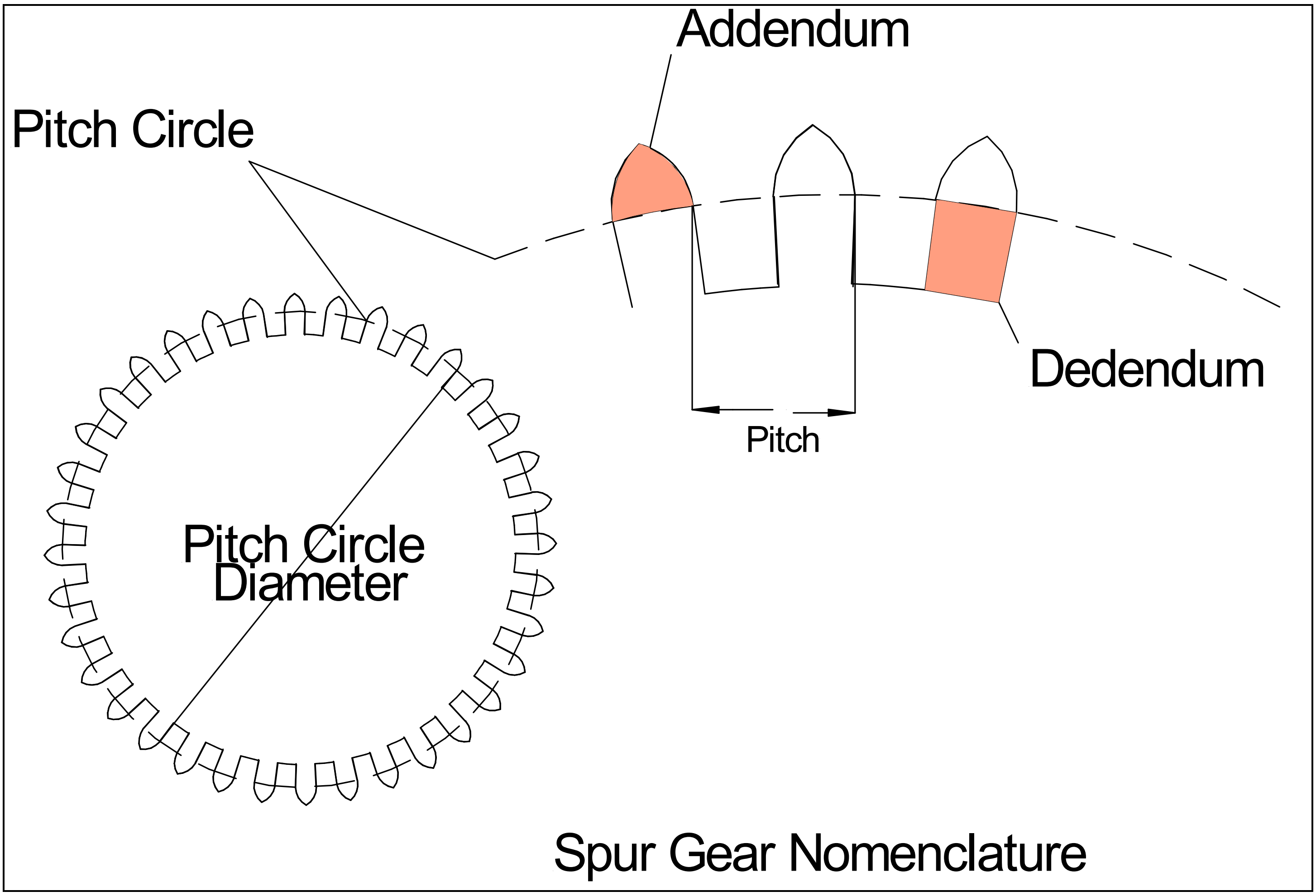
Addendum
Pitch Circle
Dedendum
Pitch
Pitch Circle
Diameter
Spur Gear Nomenclature

[1] The earliest evidence of an investigation of the problem of obtaining smooth or uniform motion from toothed gearing, that is a constant velocity ratio, relates to the work of Olaf Roemer a Danish astronomer who proposed the epicycloidal form in 1674. This work was not generally taken up. Nor was the work of Camus or Willis who later conceived the idea of interchangeable gears of epicycloidal form.

[2] This is not the place for a dissertation on velocity ratio theory. It suffices to say that the addendum of the driving gear should be an epicycloid and the dedendum of the driven gear is a matching hypocycloid (ie one having the same generating circle). The generating circle is chosen to be a size which makes the dedendum of the driven gear a straight line. The diameter of this generating circle happens to be half the pitch circle diameter of the driven gear..

[3] This gets a bit fuzzy when you recall that circular pitch is measured around the circumference of the pitch circle, and see that R is measured in a straight line. The addendum will not be tangential to the dedendum at the pitch circle. The addendum arc will intersect the flank of the tooth inside the pitch circle. But no one seems to mind.

[4] This is 5% less than the geometrically correct value and provides a clearance for engaging teeth.

[5] The radii are T/2, 2T/3, and T respectively; where T is the tooth thickness measured along the pitch circle.

[6] The constants in this table correspond of course to twice the addendum height.

[7] Thus we can see that the constant addendum system is a particular case of the BS978 system - a case where the addendum factor f is held constant at 1.35 and the addendum radius factor f_r at $\pi/2$, for all sizes of wheel.

[8] The addendum radius is 1.93M rather than the 1.57M of the earlier constant addendum series.