REVIEW REPORT: PROOF OF THE INCONSISTENCY OF MAXWELL'S ELECTRODYNAMICS...'

BY: STEFFEN KUEHN, ROBERT W. GRAY

1. Introduction

The reviewer observes that Introduction is still that of the former version, where the comments (that I copied from the cited Bohm-Ahronov article) were completely neglected. There it was stressed that:

In classical electrodynamics, the vector and scalar potentials were first introduced as a convenient mathematical aid for calculating the fields. It is true that in order to obtain a classical canonical formalism, the potentials are needed. Nevertheless, the fundamental equations of motion can always be expressed directly in terms of the fields alone. In quantum mechanics, however, the canonical formalism is necessary, and as a result, the potentials cannot be eliminated from the basic equations. Nevertheless, these equations, as well as the physical quantities, are all gauge invariant; so that it may seem that even in quantum mechanics, the potentials themselves have no independent significance.

While in Quantum Mechanics the canonical formalism (requiring potentials) is necessary, this is not the case for the classical electromagnetism, where the motions equations can be expressed with safe by the fields.

Authors didn't specify their issues with respect this fundamental article.

2. Mathematical Analysis and Proof of Inconsistency ...

Authors exploits the electromagnetic potentials Φ and $\mathbf{A}$ to re-arrange classical Maxwell equations as follows (region of space with no free charges):

$$-\nabla \cdot \nabla \Phi - \frac{\partial}{\partial t} \nabla \cdot \mathbf{A} = 0 \tag{1}$$

$$\mu_0 \mathbf{j} - \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \nabla \Phi - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \mathbf{A} = 0 \tag{2}$$

$$-q \nabla \Phi - q \frac{\partial}{\partial t} \mathbf{A} = \mathbf{F}. \tag{3}$$

In the given geometry (infinite cylinder supplied by a current on the surface) the flux density is zero outside the cylinder. After few algebra authors obtain eqn.(20):

$$\frac{\partial}{\partial t} \mathbf{F} = 0.$$

Date: Dec. 3 2019.

At the end, just before conclusion they claim:

Moreover, applying this substitution does not only transform the equations (6) and (7) into themselves, but also the equations (14), (15) and (16). This can be easily confirmed by insertion and simplification. The proof of the inconsistency of the Maxwell equations to the experimental result of the Maxwell-Lodge experiment is therefore also valid for all gauge transformations and there is no specific function Λ for which $\partial F/\partial t$ would not be zero.

This is an erroneous conclusion as the one, reported in the subsection before conclusions, where it is claimed that since $\nabla \times \mathbf{F} = 0$ and $\nabla \cdot \mathbf{F} = 0$, $\mathbf{F}$ should be harmonic or a zero function! In the sequel we would provide some basic counter-example to the authors' conclusions.

2.1. **Comments to the authors conclusions.** Let us carry on all calculations for the specified problem, by solving a trivial exercise that we could provide to a student.

2.1.1. *Integral formulation.* The symmetries for the problem allow to assume the field as directed along the z axis of the solenoid of radius a. After trivial computations we obtain the magnetostatic field:

$$\begin{cases} \mathbf{B} = \mu_0 n i(t) \mathbf{z} & \text{for } r < a \\ \mathbf{B} = 0 & \text{for } r > a, \end{cases} \tag{4}$$

being n the number of coil turns per unit length.
Now, according to the definition of vector potential in integral form:

$$\oint_{\partial\Omega} \mathbf{A} \cdot \mathbf{t} ds = \iint_{\Omega} \mathbf{B} \cdot \mathbf{n} dS, \tag{5}$$

the vector potential for the specified problem is easily drawn:

$$\begin{cases} \mathbf{A} = \mu_0 \dfrac{n\, r}{2} i(t) \hat{\theta} & \text{for } r < a \\ \mathbf{A} = \mu_0 \dfrac{n\, a^2}{2r} i(t) \hat{\theta} & \text{for } r > a, \end{cases} \tag{6}$$

In a similar way, the electric field could also be derived, as follows:

$$\begin{cases} \mathbf{E} = -\mu_0 \dfrac{n\, r}{2} \dfrac{di}{dt} \hat{\theta} & \text{for } r < a \\ \mathbf{E} = -\mu_0 \dfrac{n\, a^2}{2r} \dfrac{di}{dt} \hat{\theta} & \text{for } r > a, \end{cases} \tag{7}$$

having exploited the Faraday law or the relation between electric field and potentials (assuming $\Phi = 0$):

$$\mathbf{E} = -\nabla \Phi - \frac{\partial \mathbf{A}}{\partial t}.$$

Finally the form of the force acting on a charge into the ring conductor outside the solenoid is derived as follows:

$$\mathbf{F} = -q \mu_0 \frac{n\, a^2}{2r} \frac{di}{dt} \hat{\theta} \quad \text{for } r > a,$$

It can easily verified now that in cylindrical coordinates, the above force is both curl- and divergence-free. In fact, the only non-zero force component is along $\hat{\theta}$ with r dependence, therefore

$$\nabla \times \mathbf{F} = \frac{1}{r}\frac{\partial}{\partial r}(rF_\theta) \equiv 0.$$

In a trivial way also the divergence-free property of the force for $r > a$ can be verified. In conclusion, it's hard to conclude that force is a harmonic function or a constant or zero function. The reason is that a curl-free or div-free field in a space region doesn't imply that the field is conservative or solenoidal, without specifying the topological properties of the domain. So the above force is able to supply the charge motion inside the conductor ring.

As a further element, also eqn. (20) is not coherent, with the above derivation. In fact, the electric field (and so the force) is nonzero and time-dependent as shown in the example provided above. This comes because authors tried to work directly using the force, which in the Lorentz-Maxwell framework should be computed once fields are known, by multiplying (in the specific case) by the charge which is localized in a specific region. So, first electric field is computed in the region of interest (through a statement of equations in all space) and then multiplied by the test charge. This requires the statement of the fields equations (PDEs) in all the domain and not only outside the cylinder.

2.1.2. *differential formulation.* Authors in eqns. (15)-(20) claim to directly evaluate the force and its time derivative by only considering the region $r > a$. This is simply erroneous because it means to pretend deriving the electric field by solving the PDEs only in part of the domain! If we provide a field computation through the correct differential field formulation, no contradictions arise and all perfectly works. In fact:

$$\nabla \times \mathbf{E} = 0 \quad \text{for } r > a \tag{8}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{for } r < a \tag{9}$$

From the symmetries it follows that $\mathbf{E} = E(r)\hat{\theta}$, which yields to the following equations:

$$\frac{1}{r}\frac{d}{dr}(rE(r)) = 0 \quad \text{for } r > a \tag{10}$$

$$\frac{1}{r}\frac{d}{dr}(rE(r)) = -\mu_0 n i(t) \quad \text{for } r < a, \tag{11}$$

and then the following conclusion is drawn:

$$\begin{cases} \mathbf{E} = -\mu_0 \dfrac{n\,r}{2}\dfrac{di}{dt}\hat{\theta} & \text{for } r < a \\ \mathbf{E} = -\mu_0 \dfrac{n\,a^2}{2r}\dfrac{di}{dt}\hat{\theta} & \text{for } r > a, \end{cases} \tag{12}$$

which exactly corresponds to the results derived from the integral formulation above. Further, only now the force can be operatively computed: in the location where the charge is placed. In the specific case inside the conductor. Such a force is not only *nonzero*, but also *time varying*, despite authors conclusions. So whatever the fields or potentials are of concern, a correct fields computation provides sound conclusions also when classical mechanics is involved, and no contradictions arise since, despite quantum mechanics, the canonical formalism of mechanics is not required.

3. Conclusion

In the scientific community the issue related to the potentials is going on since long time, and several discussions can be found in the literature. Some of them also cited by authors. Here we are only asked to provide a revision to a submitted manuscript, which, unfortunately, contains technical errors and therefore doesn't support any conclusion, as shown in the basic computations provided above. This independently on any already published result.