Zero Crossing Determination by Linear Interpolation of Sampled Sinusoidal Signals

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Abstract **– Measuring and monitoring quality parameters of AC power systems requires several calculations, such the actual frequency and relative phase angles between each one of the three phases. These calculations must result in accurate values to comply with the function requirements; for synchronism function, frequency inaccuracy must be less than 0.01 Hz, and phase angle inaccuracy must be less than one degree. To perform these calculations, the zero crossing instants must be determined for each monitored signal. Conventional methods use a hardware circuit to perform these determinations resulting a high-cost system, because each monitored signal must have its own circuit. Averaging is not a good alternative due to the long time required besides there is no improvement in phase angle accuracy. The purpose of this article is to introduce a low-cost one cycle method to perform these calculations by software in a DSP using samples of the monitored signals.**

Index Terms **– Algorithms, Discrete time systems, Power systems, Frequency measurement, Phase measurement, Phase synchronization, Power distribution, Power transmission, Power quality, Sampled data systems.**

I. Introduction

In AC power energy systems, measurements of quality parameters must be performed at several network points. Some measurements are expressed by waveform time scale parameters, such the frequency and phase angle values. In a three-phase AC power system, the frequency must be the nominal frequency, 60 Hz or 50 Hz, and the phase angles must be 120 degrees. However, in an actual system, these values do not match exactly the nominal ones. The frequency and phase deviations cause error measurements in other parameters, such DC and AC RMS values, and must be accounted for accurate calculations.

In AC power transmission and distribution, the synchronism function is used to allow the connection of several AC power sources to one AC power integrated system. The synchronism function requires the sources with matched voltage amplitude, frequency and phase angles. For frequency matching, the accuracy requirement is 0.01 Hz, and for phase angles the accuracy requirement is one degree.

To determine the frequency and relative phase of AC power signals, the most used method is based one the determination of zero crossing instants, according to current standards [1] and [2]. The goal, therefore, is to determine the time-stamps of the last two zero crossings of each AC power signals, allowing calculating each frequency and relative phases. These time-stamps must be enough accurate to satisfy the measure requirements.

To reach the 0.01 Hz inaccuracy on the 60 Hz nominal frequency, the equivalent sample rate must be 6000 times the nominal frequency, that is, 360 kHz, or 2.78 μs inaccuracy in a time-stamp of the zero crossing instant. This time-stamp inaccuracy results 0.06 degree accuracy in phase angles, meeting the requirements for this measure.

II. Averaging Implementation

Averaging along several cycles of the power signals reduces the error measurements by the inverse of used amount of cycles.

When using averaging the number of cycles to reach the accuracy requirement of 0.01 Hz is equivalent to 6000 samples. In a 64 samples per nominal frequency system, it is necessary at least 94 cycles, more than 1.5 seconds to perform one measurement.

Moving averages may result in more closely spaced measurements, but the time response for a perturbation is not improved, making this method not appropriated for fast response systems.

III. Hardware Implementation

In a hardware circuit implementation, a 625 kHz clock was used to perform the time interval measurements between two consecutive zero crossing raising instants, resulting an accuracy of 0.006 Hz in frequency measurement. In the same fashion, a 39.0625 kHz clock was used to perform the time interval measurements between two zero crossing raising instants from two signals, resulting an inaccuracy of 0.6 degrees in phase difference measurement.

The input signals were filtered by 3 kHz fourth-order low-pass Butterworth filters and compared to zero value, generating square-wave input signals to digital circuits.

The digital circuits were implemented in a CPLD. The "frequency meter" uses 40 macro-cells, and the "phase meter" uses more 22 macro-cells of the CPLD, for only one pair of signals. For a second "frequency meter", it is used 32 macro-cells, resulting the use of 94 macro-cells for one implementation of three-phase synchronization function.

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IV. Interpolation Software Implementation

On the proposed method, interpolation is performed by software implementation in a DSP, using samples of the signals. The sample instant is defined by hardware, for sample time-stamp accuracy, and the DSP must perform all the calculations in less than one sample time.

A. System Requirements

When using 64 samples per nominal cycle, there is a time interval of 260 µs between the samples, which is very small for the needed accuracy. By interpolation between the trailing and leading samples of the zero crossing instant, this accuracy may be improved, reaching the needed value. This final accuracy depends on the time-stamp precision of the used samples, the time interval between the samples, and the amount of digitalization steps between the sample values.

The time-stamp precision of the used samples defines the reference time skew for the calculation result. Implemented by hardware, this sample time-stamp precision is very accurate, a few nano-seconds, and may be worthless.

The time interval between samples defines the initial accuracy for the calculation of zero crossing time stamp. This time interval is 260 µs for the implemented system.

The amount of digitalization steps between the sample values defines the improvement in the initial accuracy to result the final accuracy. For the needed accuracy of 2,78 µs in time-stamp of zero crossing calculations, the minimum amount of digitalization steps between the used samples is 94. By using 14-bit signaled analog to digital conversion, and signal amplitude 50% the maximum value, there are at least 400 digitalization steps between two samples near the zero crossing, exceeding the minimum needed.

B. Linear Interpolation

For speed, it was implemented a linear interpolation. The shape of the signal is very close the straight line near the zero crossing.

Near the zero crossing, the samples may be described as points in a straight line, defined by the angular, a, and linear, b, parameters. The x axis may be expressed in sample units, and two consecutive samples may be expressed by (1).

$$\begin{aligned} x_n &= a \times n + b \\ x_{n+1} &= a \times (n+1) + b \end{aligned} \tag{1}$$

The parameters as a function of samples are given by (2).

$$\begin{aligned} a &= x_{n+1} - x_n \\ b &= (n+1) \times x_n - n \times x_{n+1} \end{aligned} \tag{2}$$

The choice of consecutive used samples must be that the raising zero crossing is between that ones, satisfying (3).

$$x_n \le 0 < x_{n+1} \tag{3}$$

Defining a d sample displacement for interpolation, the interpolated value at the $n+d$ is given by (4).

$$x_{n+d} = a \times (n+d) + b \tag{4}$$

The desired instant $n+d$ of the zero crossing is calculated by making the interpolated value equal to zero. The result is given by (5).

$$n + d = -\frac{b}{a} = \frac{n \times x_{n+1} - (n+1) \times x_n}{x_{n+1} - x_n} \tag{5}$$

The displacement d is given by (6).

$$d = \frac{-x_n}{x_{n+1} - x_n} \tag{6}$$

Due to (3), $0 \le d < 1$ is the fractional component of the zero crossing instant in sample numbers $n+d$.

The samples x_n and x_{n+1} are respectively the trailing and leading samples from the zero crossing instant.

C. Improvement by the Least Square Method

Generally, there is a noise added to the input signal samples, making not reliable the calculations over only one pair of consecutive samples.

A sequence of N consecutive samples, $N/2$ trailing and $N/2$ leading the supposed zero crossing instant, may define a vector X of samples, as in (7).

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_N \end{bmatrix} \tag{7}$$

These samples must be adjusted to a straight line, defined by (8), that defines the adjusted samples.

$$\hat{x}_n = a \times n + b \tag{8}$$

The Least Square Method minimizes the sum S of squares of differences between the samples and the respective adjusted sample, the expression in (9).

$$S = \sum_{i=1}^{N} (x_i - \hat{x}_i)^2 \tag{9}$$

The sample vector X in (7) may be expressed by a set of values approximated by the expression in (8), as shown in (10).

$$\begin{aligned} x_1 &\cong a + b \\ x_2 &\cong 2 \times a + b \\ &\dots \\ x_N &\cong N \times a + b \end{aligned} \tag{10}$$

A linear equation system may be written by equaling the samples to the adjusted samples, as in (11).

$$\begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_N \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ \dots & \dots \\ N & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} \tag{11}$$

This linear system is in the form $[Y]=[A][X]$. By the least square method, both sides are multiplied by $[A]^t$, as shown in (12).

$$\begin{bmatrix} 1 & 2 & \dots & N \\ 1 & 1 & \dots & 1 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_N \end{bmatrix} = \begin{bmatrix} 1 & 2 & \dots & N \\ 1 & 1 & \dots & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ \dots & \dots \\ N & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} \tag{12}$$

The resulting equation is sown in (13).

$$\begin{bmatrix} \sum_{i=1}^{N} (i \times x_i) \\ \sum_{i=1}^{N} x_i \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} i^2 & \sum_{i=1}^{N} i \\ \sum_{i=1}^{N} i & N \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} \tag{13}$$

By using (5) in (13), the desired value $n+d$ is obtained. The expression is shown in (14).

$$n + d = -\frac{b}{a} = \frac{\sum_{i=1}^{N} i^2 \times \sum_{i=1}^{N} x_i - \sum_{i=1}^{N} i \times \sum_{i=1}^{N} (i \times x_i)}{\sum_{i=1}^{N} i \times \sum_{i=1}^{N} x_i - N \times \sum_{i=1}^{N} (i \times x_i)} \tag{14}$$

D. DSP Application

In an implemented example, N was chosen to 8. The value 8 is sufficiently great to reduce the noise errors, and not sufficient to introduce errors due to the linear approximation of a sinusoidal signal with 64 samples per cycle. The far sample from the zero crossing instant is 22,5 degrees far, resulting an error of 2,5% in the individual sample approximation.

With $N=8$, (14) becomes the expression in (15).

$$n + d = \frac{42x_1 + 33x_2 + 24x_3 + 15x_4 + 6x_5 - 3x_6 - 12x_7 - 21x_8}{7x_1 + 5x_2 + 3x_3 + x_4 - x_5 - 3x_6 - 5x_7 - 7x_8} \tag{15}$$

Equation (15) may be implemented in a DSP by using two scalar products of vectors and one division, as shown in (16).

$$n + d = \frac{[B] \times [X]}{[A] \times [X]} \tag{16}$$

Vector $[X]$ is defined in (7). Vectors $[A]$ and $[B]$ are defined respectively in (17) and (18).

$$[A] = [\,7 \quad 5 \quad 3 \quad 1 \quad -1 \quad -3 \quad -5 \quad -7\,] \tag{17}$$

$$[B] = [\,42 \quad 33 \quad 24 \quad 15 \quad 6 \quad -3 \quad -12 \quad -21\,] \tag{18}$$

A good choice of the used samples makes $n = 4$, and d may be obtained by subtracting the value *4* from (15), resulting the vector $[B]$ given by (19).

$$[B] = [\,14 \quad 13 \quad 12 \quad 11 \quad 10 \quad 9 \quad 8 \quad 7\,] \tag{19}$$

Thus, $n+d$ may be replaced by d in (16). The same good choice makes $0 \le d < 1$, as the fractional component of the zero crossing instant in sample numbers.

To comply with a Q15 number representation standard, which uses the interval [-1,1), the values of samples and coefficients in vectors $[A]$ and $[B]$ must be normalized. The samples were normalized to the maximum amplitude, which matches the value one, while the coefficients were all divided by the value 16, which is a more adequate power of two to comply with Q15 standard. New vectors $[A]$ and $[B]$ are respectively shown in (20) and (21).

$$[A] = \begin{bmatrix} \frac{7}{16} & \frac{5}{16} & \frac{3}{16} & \frac{1}{16} & -\frac{1}{16} & -\frac{3}{16} & -\frac{5}{16} & -\frac{7}{16} \end{bmatrix} \tag{20}$$

$$[B] = \begin{bmatrix} \frac{14}{16} & \frac{13}{16} & \frac{12}{16} & \frac{11}{16} & \frac{10}{16} & \frac{9}{16} & \frac{8}{16} & \frac{7}{16} \end{bmatrix} \tag{21}$$

E. Implemented Algorithm

The implemented algorithm obtains the $N = 8$ consecutive samples to be used in calculations, and performs (16) using (20) and (21), obtaining d which satisfy $0 \le d < 1$.

First, the algorithm determines that the last $N = 8$ samples are all in the negative half cycle.

Second, the algorithm selects the most adequate set of consecutive samples, which is when the amount of positive samples are the half of them, that is, $N/2 = 4$.

The chosen sets of samples are used to perform the mathematical equations that result the desired zero crossing instant d.

V. Results

The presented method was implemented in a Texas Instruments TMS320C5407 DSP, and applied to several AC power lines to determine the zero crossing instants, at different times. Two examples are shown here.

The shown sampled signals were collected from the actual power line with 64 samples per nominal cycle and analyzed by the presented method. Fig. 1 shows one cycle of the first signal, the chosen samples used in calculations and the calculated zero crossing point. Fig. 2 shows the same signal, amplified to show only the interval of used samples. Fig. 3 and fig. 4 show the same for the second signal.

Note the visual accuracy of the presented method, in the first signal. In the second signal, there is a correction due to the noise in the nearest samples from the zero crossing.

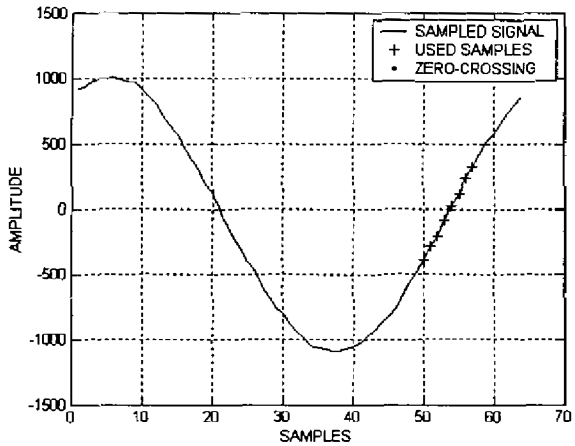

Fig. 1. Zero crossing calculations for the first signal

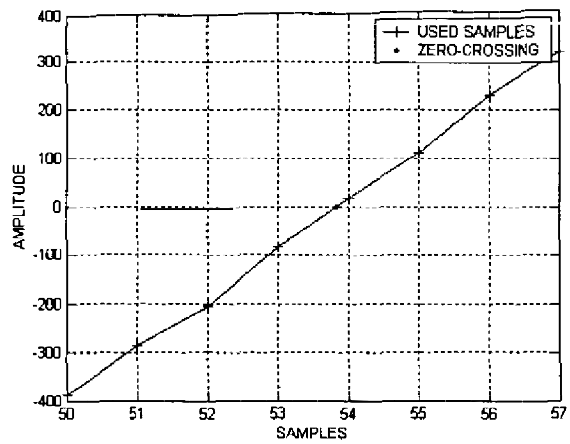

Fig. 2. Zero crossing calculations for the first signal - detailed

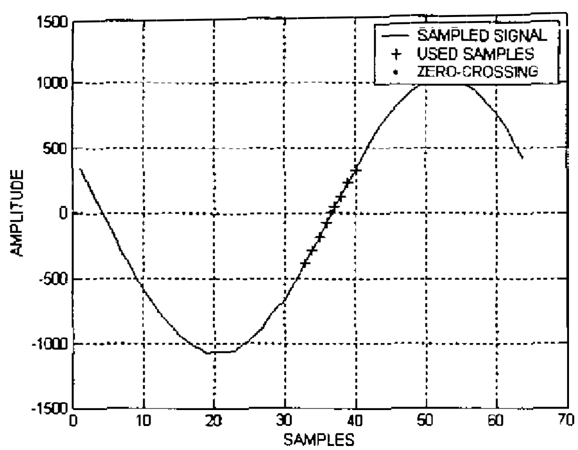

Fig. 3. Zero crossing calculations for the second signal

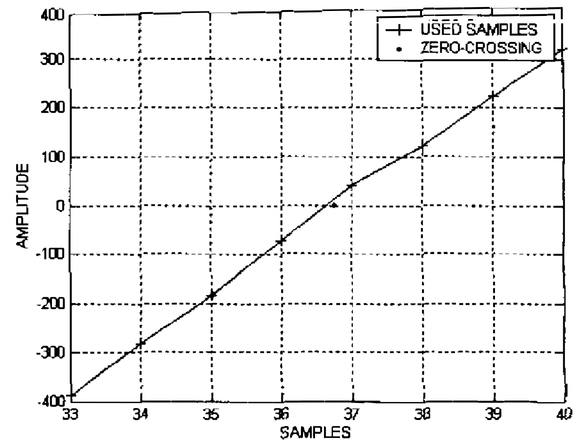

Fig. 4. Zero crossing calculations for the second signal - detailed

VI. Conclusions

A method for estimating the zero crossing instant of sampled sinusoidal signal using linear interpolation was presented. This method provides faster results than the averaging approach. When several AC power signals are to be monitored a dedicated hardware approach is more expensive than the proposed method where the calculations for all signals are performed in one DSP, instead of individual frequency meter digital circuits. It also allows system flexibility, for determining the relative phases between any pair of monitored signals. By the calculations presented, the accuracy is sufficient to satisfy the requirements of the synchronization function.

Plots of the interpolated signals under different noise conditions showed the presented method can well determine the zero crossing instant. The use of a linear least square fit with 8 samples resulted in more robust result than the use of only two samples (leading and trailing) for higher noise conditions.

This method is sought to be very efficient in systems with several AC signals, from various power lines, even at different frequencies, given more flexibility in system design.

VII. References

[1] *Instrumentation and Measurement Group of the IEEE Power Engineering Society, USA. IEEE standard definitions of physical quantities for fundamental frequency and time metrology* - IEEE Std. 1139-1988, Apr. 28, 1989.

[2] *IEEE Standards Coordinating Committee 27 on Time and Frequency, USA. IEEE standard definitions of physical quantities for fundamental frequency and time metrology - random instabilities* - IEEE Std. 1139-1999.