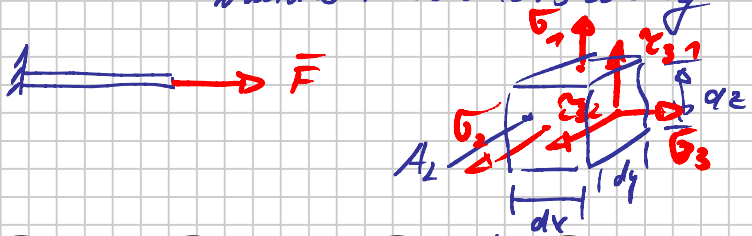
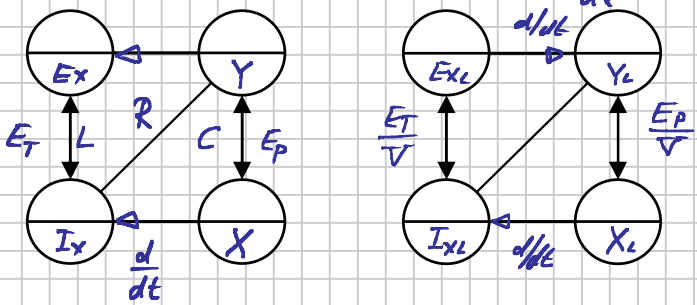
Wandler in Felddarstellung





lokale Größen

$X_L := \frac{X}{A_L}$ Flächendichte

Was ist die Speichergröße?

$\frac{E}{V} \sim Y_L = \frac{Y}{l}$

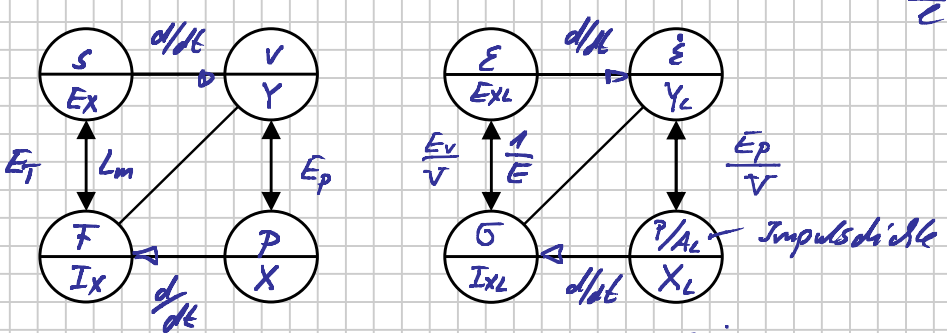
$E_{xL} = \frac{E_x}{l}$

A_L: lokales Flächenelement

l: Bezugslänge

Bsp.: Mechanik (Impuls)

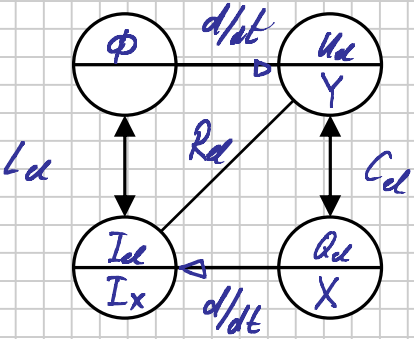
NR: $\frac{s}{l} = \frac{\Delta l}{l} = \varepsilon$

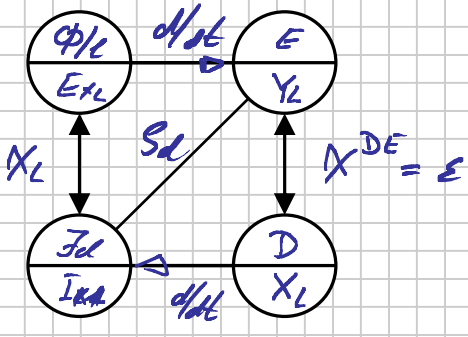


Suszeptibilitäten $\quad \chi := \frac{\partial q}{\partial i} = \chi^{qi}$

global	lokal
$\chi^{SF} = \frac{\partial s}{\partial F} \Rightarrow \frac{s}{F} = n = L_m$	$\chi^{\varepsilon\sigma} = \frac{\partial \varepsilon}{\partial \sigma}$ $\quad \frac{\varepsilon}{\sigma} = \frac{1}{E}$ (Hookesches Ges
	$\chi^{\varepsilon\sigma} = \frac{1}{E}$ $\quad \sigma = E \cdot \varepsilon$
$\chi^{pv} = \frac{\partial p}{\partial v} = m_T$	$\chi^{p/A_L, \dot{\varepsilon}} = \frac{\partial p/A_L}{\partial \dot{\varepsilon}} = ?$

Bsp.: Elektrotechnik





$D = \frac{\partial Q_{el}}{\partial A_L}$ elektr. Flussdichte

$J_{el} = \frac{\partial I_{el}}{\partial A_L}$ Stromdichte

$E = \frac{\partial U_{el}}{\partial L}$ elektr. Feldstärke

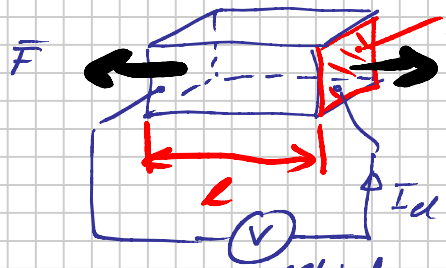
$\chi^{DE} = \varepsilon = \varepsilon_0 \cdot \varepsilon_r$ Permittivität

S_{el} = spez. elektr. Widerstand

Anmerk: $\chi^{DE} = \frac{\partial D}{\partial E} = \varepsilon$

$\chi_e = \frac{\chi^{DE}}{\varepsilon_0} = \varepsilon_r$

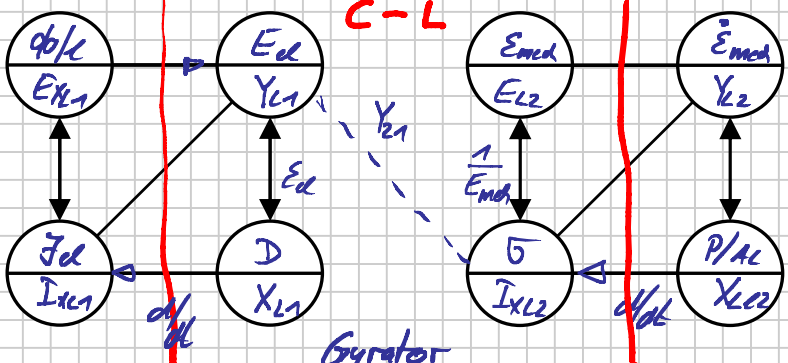
Der Piezoelektrische Wandler



Kopplung elektrisch – mechanisch

elektrisch-mechanisch

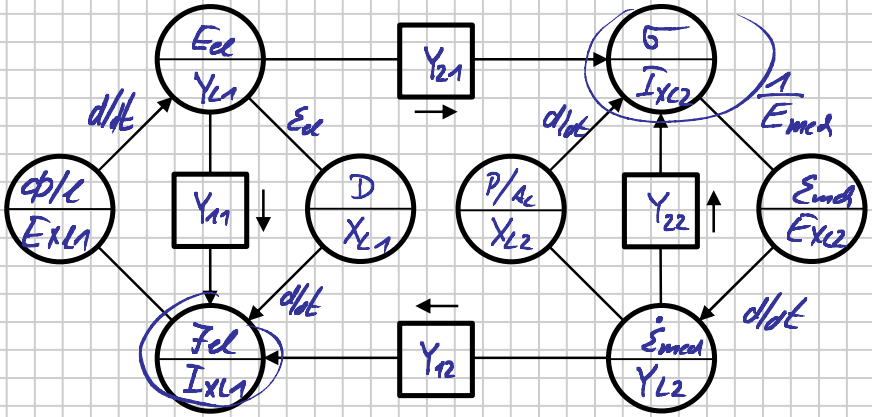
C-L



Gyrator

$Y_{21} = Y_{12} = \left(\frac{\partial \sigma}{\partial E_{el}}\right)_D$

$E_{el} = \frac{1}{\varepsilon_{el}} \cdot D$



$$F_{el} = Y_{11} \cdot E_{el} + Y_{12} \cdot \dot{\varepsilon}_{mech} \quad (I)$$

$$\sigma = Y_{21} \cdot E_{el} + Y_{22} \cdot \dot{\varepsilon}_{mech} \quad (II)$$

Reziprozität $Y_{12} = Y_{21}$

II Mechanik

$$\sigma = Y_{21} \cdot E_{el} + Y_{22} \cdot \dot{\varepsilon}_{mech}$$

Vor: keine Verluste $\eta = 1 \rightsquigarrow Y_{22} = 0$

$$\sigma = Y_{21} \cdot E_{el} + \varepsilon_{mech} \cdot E_{mech}$$

(Hookesches Gesetz $\sigma = \varepsilon \cdot E$)

UR: $\chi^{E\sigma} = \frac{\varepsilon}{\sigma} = \frac{1}{E_{mech}} = \frac{\Delta l}{l} \cdot \frac{A}{F}$

$E_{mech} = \frac{l}{A} \cdot \frac{F}{\Delta l} = \frac{l}{A} \cdot \frac{1}{L_m}$

$$\sigma = Y_{21} \cdot E_{el} + \varepsilon_{mech} \cdot \frac{l}{A} \cdot \frac{1}{L_m} \quad | \cdot A$$

UR: $\sigma = \frac{F}{A}$; $E_{el} = \frac{U_{el}}{l}$

$$F = Y_{21} \cdot \frac{A}{l} \cdot U_{el} + \varepsilon_{mech} \cdot l \cdot \frac{1}{L_m}$$

$\frac{\Delta l}{l} \cdot l \cdot \frac{1}{L_m} = \Delta l \cdot \frac{1}{L_m}$

$V = \frac{d}{dt} \Delta l$; $j\omega \Delta l$ $\quad \Delta l = \frac{1}{j\omega} \cdot V$

$$F = Y_{21} \cdot \frac{A}{l} \cdot U_{el} + \frac{1}{j\omega L_m} \cdot V \quad (II)$$

I. Elektrotechnik

$$F_{el} = Y_{11} \cdot E_{el} + Y_{12} \cdot \dot{\varepsilon}_{mech}$$

Vor: keine Verluste $\eta = 1$ $Y_{11} = 0$

NB: $Y_{12} \cdot \dot{\varepsilon}_{mech} = Y_{12} \cdot \frac{d}{dt}\left(\frac{\Delta l}{l}\right) = Y_{12} \cdot \frac{1}{l} \cdot v$

$D = \varepsilon_{el} \cdot E_{el}$; $J_{el} = \dot{D} = \frac{d}{dt}\left(\varepsilon_{el} \cdot E_{el}\right)$

$J_{el} = \frac{d}{dt}\left(\varepsilon_{el} \cdot E_{el}\right) + Y_{12} \cdot \frac{1}{l} \cdot v \quad | \cdot A$

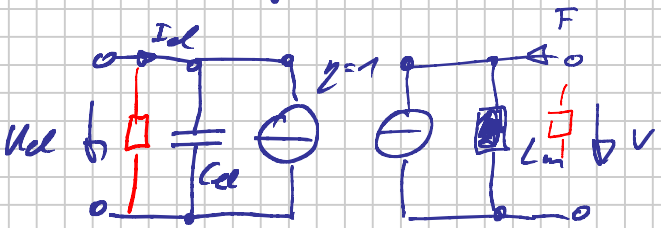
$I_{el} = \frac{d}{dt}\left(\varepsilon_{el} \cdot E_{el}\right) \cdot A + Y_{12} \cdot \frac{A}{l} \cdot v \quad | \quad E_{el} = \frac{U_{el}}{l}$

$I_{el} = \frac{d}{dt}\left(\varepsilon_{el} \cdot \frac{A}{l} \cdot U_{el}\right) + Y_{12} \cdot \frac{A}{l} \cdot v$

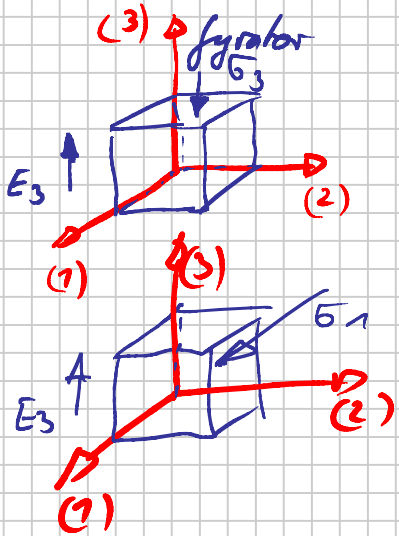
$$I_{el} = j\omega C_{el} \cdot U_{el} + Y_{12} \cdot \frac{A}{l} \cdot v \qquad \text{I.}$$

$Y_{12} = Y_{21} =: e$ (Piezomodul)

$$\begin{bmatrix} I_{el} \\ F \end{bmatrix} = \begin{bmatrix} j\omega C_{el} & e \cdot \frac{A}{l} \\ e \cdot \frac{A}{l} & \frac{1}{j\omega L_m} \end{bmatrix} \cdot \begin{bmatrix} U_{el} \\ v \end{bmatrix}$$



$Y_{12} = Y_{21} = e \cdot \frac{A}{l}$



$e = e_{33} > 0$ (Längseffekt)

$e = e_{31} < 0$ (Quereffekt)

Index 1: elektr. K.
Index 2: mech. K.

Anmerkungen

TD-Tensorform

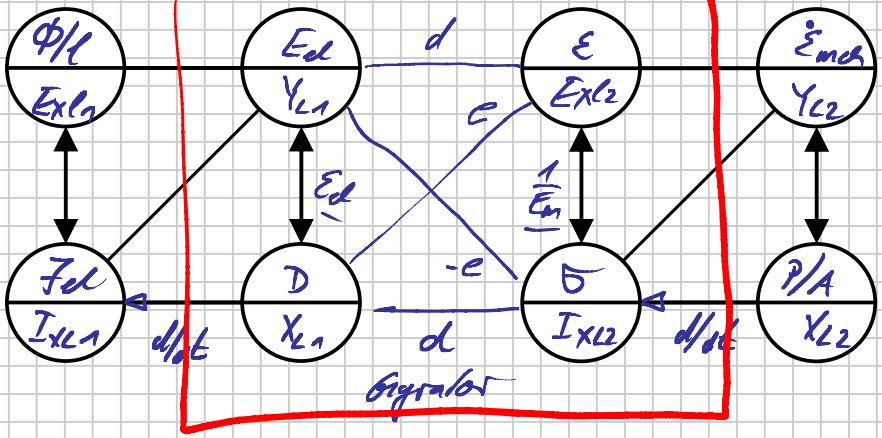
$$D = \varepsilon \cdot E + e \cdot S \qquad D = \varepsilon_{el} \cdot E_{el} + e \cdot \varepsilon_{mech}$$

$$T = c \cdot S - e \cdot E \qquad \sigma = E_{mech} \cdot \varepsilon_{mech} - e \cdot E_{el}$$

SD-Tensorform

$$S = s \cdot T + d \cdot E \qquad \varepsilon_{mech} = \frac{1}{E_{mech}} \cdot \sigma + d \cdot E_{el}$$

$$D = d \cdot T + \varepsilon \cdot E \qquad D = d \cdot \sigma + \varepsilon_{el} \cdot E_{el}$$



(1) $X := \frac{\partial q}{\partial i}$ $\quad E_{el} = i_T$; $\sigma = i_p$ → keine Suszeptibilität

$D = q_p$; $\varepsilon_{mech} = q_T$ → " "

(2) $E_{el} = i_T$; $\varepsilon_m = q_T$ $\quad X^{\varepsilon_m E_e} = \frac{\partial \varepsilon_m}{\partial E_e} = d$

$D = q_p$; $\sigma = i_p$ $\quad X^{D\sigma} = \frac{\partial D}{\partial \sigma} = d$