Theory of a Speaker in a Vented Box

Introduction

The aim of this document is to describe the way an electrodynamic loudspeaker works in a vented cabin. The voice coil of the speaker is driven by the voltage produced by an amplifier. The front side of the membrane is loaded by the radiation impedance of a spherical wave, the rear side works against the stiffness of the air in the cabin and the lumped mass in the port. Stiffness and mass together are forming a Helmholtz resonator that is exited by the speaker. The front side of the port radiates into free air as well. The overall power radiated from the cabin depends on the total volume velocity created by membrane and port.

Theory

Figure 1 shows the electromechanical equivalent circuit diagram of a speaker in a vented box. We will describe the parts below.

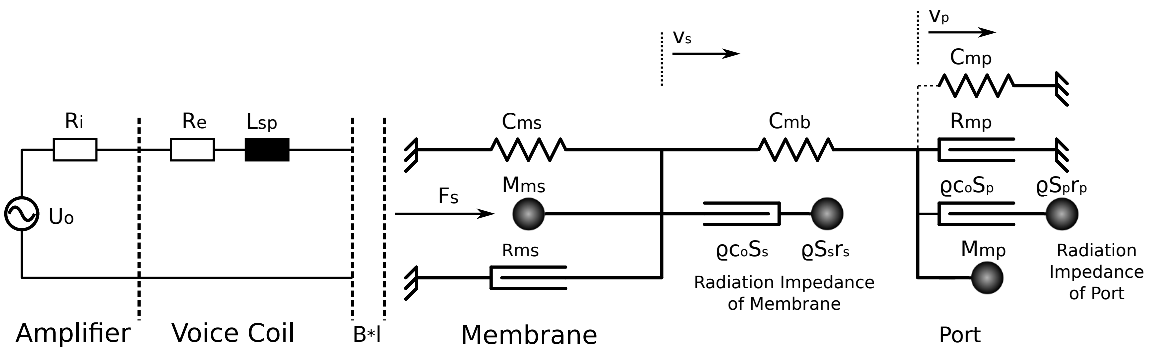

Fig. 1: Sketch of the whole system including amplifier, speaker, vented cabin and radiation impedance at front side of speaker and port.

The following terms are used to describe the components:

U_0	driving voltage of the amplifier
U_{ind}	voltage induced in the voice coil due to the movement of the membrane
R_i :	inner resistance of amplifier
R_e :	resistance of voice coil
L_{sp} :	inductivity of voice coil
R_{ms}, M_{ms}, C_{ms} :	mechanical losses, mass and compliance of the membrane
C_{mb} :	compliance of air in the cabin
R_{mp}, M_{mp}, C_{mp} :	mechanical losses, mass and compliance of the port
$\rho c_o S_s$, $\rho S_s r_s$:	radiation resistance and acoustic reactance of membrane with S_s area of membrane and $r_s = \sqrt{S_s/\Omega}$ with Ω solid angle of radiation
$\rho c_o S_p$, $\rho S_p r_p$:	radiation resistance and acoustic reactance of port
Bl :	The product of magnet field strength in the voice coil gap and the length of wire in the magnetic field, in tesla-meters (Tm)
v_s	velocity of membrane
v_p	velocity of port

Describing Equations

Voltage U induced in the voice coil by membrane velocity v_s:

$$U_{ind} = Bl \cdot v_s \tag{1}$$

Force F_s driving the membrane due to the voice coil current I:

$$F_s = Bl \cdot I \tag{2}$$

Mechanical impedance Z_m:

$$Z_m = F_s / v_s \tag{3}$$

For a monopole source the pressure at the microphone location depending on solid angle of radiation Ω, angular frequency ω, density ρ, volume velocity of membrane $v_s S_s$ and port $v_p S_p$, angular wave number $k = \omega / c$ and the radius vector r could be described as:

$$p = \frac{j\omega\rho}{\Omega} \left(v_s S_s + v_p S_p \right) \frac{e^{-jkr}}{r} \tag{4}$$

The stiffness of the cabin (defined as the inverse of the compliance)

$$\frac{1}{C_{mb}} = \frac{\varphi c^2 S^2}{V} \tag{5}$$

valid for the area S of membrane S_s as well as for the port S_p (both are working against the same spring).

Velocity of Membrane depending on Electrical and Mechanical Impedance and on Driving Voltage of Voice Coil

The total mechanical impedance of the speaker and the load on both sides of the membrane Z_m can be expressed as an additional electrical impedance Z_{em} in series with that of the voice coile, coupled via the magnetic field. To describe it we are using the two coupling equations 1 and 2.

$$Z_{em} = \frac{U_{ind}}{I} = \frac{(Bl)^2}{Z_m} \tag{6}$$

The pure electrical part of the impedance is given by

$$Z_{el} = R_i + R_e + i\omega L_{sp} \tag{7}$$

thus the overall impedance seen by the source (driving amplifier) could be written as:

$$\frac{U_0}{I} = Z_{el} + Z_{em} = Z_{el} + \frac{(Bl)^2}{Z_m} \tag{8}$$

Using equation 3 and 2 we can derive the dependency of the velocity of the membrane v_s from the driving voltage U_0:

$$v_s = \frac{F_s}{Z_m} = \frac{I \cdot Bl}{Z_m} = \frac{U_0 Bl}{Z_m (Z_{el} + \frac{(Bl)^2}{Z_m})} = \frac{U_0 Bl}{Z_m Z_{el} + (Bl)^2} \tag{9}$$

Total Mechanical Impedance of Radiating Speaker Membrane

Now we have to find the equation for the mechanical impedance depending on the parameters of the speaker, the compliance of the cabin, the port and the radiation impedance.

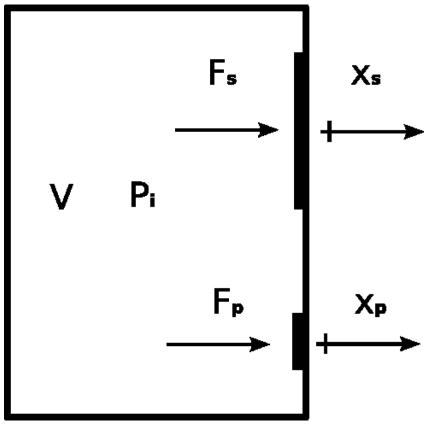

Fig. 2: Vented speaker cabin with speaker membrane and port with V denotes volume, $P_i$pressure inside cabin, F and x force at and auslenkung of membrane and port.

First we derive the impedance F_s/v_s at the rear side of the membrane. For harmonic motion the velocity $\widetilde{v}$ is proportional to the displacement $\widetilde{x}$ with factor $\widetilde{v} = i\omega\widetilde{x}$. The mechanical impedance of the stiffness of the rear volume is

$$Z_{mb} = \frac{1}{i\omega C_{mb}}, \tag{10}$$

the impedance of the port is

$$Z_{mp} = i\omega M_{mp} + \frac{1}{i\omega C_{mp}} + R_{mp} + \frac{1}{\frac{1}{\rho c_0 S_p} + \frac{1}{i\omega\rho S_p r_p}} \tag{11}$$

including a compliance C_{mp} and a resistance R_{mp} that we normally neglect. Finally the impedance of the front side of the membrane is

$$Z_{ms} = i\omega M_{ms} + \frac{1}{i\omega C_{ms}} + R_{ms} + \frac{1}{\frac{1}{\rho c_0 S_s} + \frac{1}{i\omega\rho S_s r_s}}. \tag{12}$$

For a descripion of the last summand see the appendix. The pressure in the cabin P_i exerts a force to membrane and port

$$P_i = \frac{F_s}{S_s} = \frac{F_p}{S_p}. \tag{13}$$

This inner pressure is created by displacement of membrane and port. With $P_i = F/S = x/(C_{mb}S)$, $x = v/(i\omega)$ and equation 5 we can write

$$P_i = \frac{\varphi c^2 S_s}{V} x_s + \frac{\varphi c^2 S_p}{V} x_p = \frac{1}{C_{mb}S_s}\left[x_s + \frac{S_p}{S_s}x_p\right] = \frac{1}{i\omega}\frac{1}{C_{mb}S_s}\left[v_s + \frac{S_p}{S_s}v_p\right] \tag{14}$$

and thus

$$F_s = \frac{1}{i\omega}\frac{1}{C_{mb}}\left[v_s + \frac{S_p}{S_s}v_p\right]. \tag{15}$$

Using $v_p = F_p/Z_{mp} = F_s S_p/(S_s Z_{mp})$ and a few conversions to bring F_s/v_s on the left side, we finally get the total mechanical impedance of the rear side of the membrane

$$\frac{F_s}{v_s}(rear) = \frac{1}{\frac{1}{Z_{mb}} - \frac{1}{Z_{mp}}\left(\frac{S_p}{S_s}\right)^2} \tag{16}$$

and thus the overall impedance of front and rear side of the membrane is given by

$$Z_m = Z_{ms} + \frac{1}{\frac{1}{Z_{mb}} - \frac{1}{Z_{mp}}\left(\frac{S_p}{S_s}\right)^2} \tag{17}$$

Velocity of Port as a Function of the Velocity of the Membrane

In the next step we'd like to express the velocity of the port by the velocity of the membrane. Using eq. 16 we can write

$$v_p = \frac{F_p}{Z_{mp}} = \frac{S_p}{S_s}\frac{F_s}{Z_{mp}} = \frac{S_p}{S_s}\frac{1}{Z_{mp}}\frac{1}{\frac{1}{Z_{mb}} - \frac{1}{Z_{mp}}\left(\frac{S_p}{S_s}\right)^2} v_s \tag{18}$$

and finally

$$v_p = Z_{mb}\frac{1}{\frac{S_s}{S_p}Z_{mp} + \frac{S_p}{S_s}Z_{mb}} v_s \tag{19}$$

Total Volume Velocity

To calculate the radiated sound pressure level (eq. 4) we have to calculate the over all volume velocity Q of membrane and port:

$$Q = S_s v_s + S_p v_p = \left(S_s + S_p Z_{mb} \frac{1}{\frac{S_s}{S_p} Z_{mp} + \frac{S_p}{S_s} Z_{mb}} \right) v_s \tag{20}$$

and thus

$$Q = \left(1 - \frac{Z_{mb}}{(\frac{S_s}{S_p})^2 Z_{mp} + Z_{mb}} \right) S_s v_s \tag{21}$$

Finally to get the SPL at a microphone location we substitute above Q and v_s from eq. 9 into eq.yx 4.

Thiele & Small Parameters

Most of the time you won't get the physical parameters of a loudspeaker directly, instead you will find it's T&S parameters. To get an idea how those two are connected I copied the following text mainly from Wikipedia (see http://en.wikipedia.org/wiki/Thiele/Small).

Fundamental small signal mechanical parameters

These are the physical parameters of a loudspeaker driver, as measured at small signal levels, used in the equivalent electrical circuit models. Some of these values are neither easy nor convenient to measure in a finished loudspeaker driver, so when designing speakers using existing drive units (which is almost always the case), the more easily measured parameters listed under Small Signal Parameters are more practical.

- S_d- Projected area of the driver diaphragm, in square metres.
- M_{ms}- Mass of the diaphragm/coil, including acoustic load, in kilograms. Mass of the diaphragm/coil alone is known as M_{md}(!!! in the above text M_{ms} is taken as the mass of the diaphragm/coil, excluding acoustic load !!!)
- C_{ms}- Compliance of the driver's suspension, in metres per newton (the reciprocal of its 'stiffness').
- R_{ms}- The mechanical resistance of a driver's suspension (ie, 'lossiness') in N·s/m
- L_e- Voice coil inductance measured in millihenries (mH) (Frequency dependent, usually measured at 1 kHz).
- R_e- DC resistance of the voice coil, measured in ohms.
- Bl- The product of magnet field strength in the voice coil gap and the length of wire in the magnetic field, in tesla-metres (T·m).

Small signal parameters

These values can be determined by measuring the input impedance of the driver, near the resonance frequency, at small input levels for which the mechanical behavior of the driver is effectively linear (ie, proportional to its input). These values are more easily measured than the fundamental ones above.

- F_s– Resonance frequency of the driver
  $F_\mathrm{s} = \frac{1}{2\pi \cdot \sqrt{C_\mathrm{ms} \cdot M_\mathrm{ms}}}$
- Q_{es}– Electrical Q of the driver at F_s
  $Q_\mathrm{es} = \frac{2\pi \cdot F_\mathrm{s} \cdot M_\mathrm{ms} \cdot R_\mathrm{e}}{(Bl)^2}$
- Q_{ms}– Mechanical Q of the driver at F_s
  $Q_\mathrm{ms} = \frac{2\pi \cdot F_\mathrm{s} \cdot M_\mathrm{ms}}{R_\mathrm{ms}}$

- Q_{ts}– Total Q of the driver at F_s
  $Q_{\mathrm{ts}} = \frac{Q_{\mathrm{ms}} \cdot Q_{\mathrm{es}}}{Q_{\mathrm{ms}} + Q_{\mathrm{es}}}$
- V_{as}– Equivalent Compliance Volume, i.e. the volume of air which, when acted upon by a piston of area S_d, has the same compliance as the driver's suspension:
  $V_{\mathrm{as}} = \rho \cdot c^2 \cdot S_{\mathrm{d}}^2 \cdot C_{\mathrm{ms}}$
  where ρ is the density of air (1.184 kg/m3 at 25 °C), and c is the speed of sound (346.1 m/s at 25 °C). Using SI units, the result will be in cubic meters. To get V_{as} in litres, multiply by 1000.

It follows a short list how the T&S parameters are calculated back into their physical equivalents inside the calculation program *box*:

$C_{ms} = V_{as}/(1000 \cdot c_0{}^{\mathbf{2}} \cdot \rho \cdot S_s)$
$M_{ms} = 1/((2\pi f_s)^2 C_{ms})$
$R_{ms} = 1/(2\pi f_s C_{ms} Q_{ms})$
$Bl = \sqrt{(2\pi f_s R_e M_{ms}/Q_{es})}$
$C_{mb} = V_b/(1000 \cdot c_0{}^{\mathbf{2}} \rho S_s{}^{\mathbf{2}})$
$M_{mp} = 1/(2\pi f_b)^{\mathbf{2}}/C_{mp}$
$M_{mp} = (S_p/(2\pi f_b S_s))^{\mathbf{2}}/C_{mb}$

Appendix

How to come from the impedance of a spehrical wave to the equivalent circuit diagram?

The acoustical impedance of a spherical wave is given by:

$$Z_a = \frac{p}{v} = \rho c \frac{ikr}{1 + ikr} \tag{22}$$

we have to multiply by the area of the membrane (port) to get the mechanical impedance:

$$Z_m = \frac{pS}{v} = \frac{F}{v} = \rho c S \frac{ikr}{1 + ikr} = \frac{1}{\frac{1}{i\omega\rho r S} + \frac{1}{\rho c S}} \tag{23}$$

The latter equation describes a series connection of radiation resistance $\rho c S$ and acoustic reactance $\rho r S$ in the impedance analogy (http://en.wikibooks.org/wiki/Engineering_Acoustics/Electro-Mechanical_Analogies)

Errata

- Changed wrong Formula $M_{mp} = 1/(2\pi f_b)^{\mathbf{2}}/C_{mb}$ to $M_{mp} = 1/(2\pi f_b)^{\mathbf{2}}/C_{mp}$ on 03/21/2012