Simplified Q Multiplier

Portion of cathode-follower output is stepped up by passive components and fed back to grid of tube to give extremely high selectivity with absolute stability. Extra parts needed are one tube, one capacitor and two resistors

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With the rise of radio navigation, c-w radar, and other systems requiring maximum signal-to-noise ratio, there have grown up in recent years a large number of applications for amplifying systems of very narrow bandwidth. Since the basic limitation on the narrowness of bandwidth which can be obtained in an ordinary tuned amplifier is the resistance associated with the tuned circuit it uses, it seems logical that one solution to the problem would be the use of an active network to supply energy to the system according to exactly the same laws by which the resistance dissipates it, so that some of the effect would be cancelled out.

The use of such active networks, known, for obvious reasons, as negative resistances, turns out to be an entirely practical method of raising the Q of a tuned circuit. Ohm's law holds exactly for a negative resistance element, except for sign change, so it is possible to treat it exactly as any other circuit component, even to the extent of combining it with the positive resistances in the circuit.

Consider, for instance, a tuned circuit having an equivalent parallel resistance R. The initial value for Q would be

$$Q_0 = \frac{R}{\omega L} \qquad (1)$$

and suppose there is put in parallel with this tuned circuit an active network having a negative resistance characteristic. The negative resistance can be combined with the positive resistance of the circuit by the usual laws of combination of parallel resistances to give the following relationship for R_{eff}:

$$R_{eff} = \frac{(-R_n)\,R}{(-R_n) + R} = \frac{R\,R_n}{R_n - R} \qquad (2)$$

which is obviously greater than the original R, corresponding to a multiplication of the original Q by a factor equal to the ratio of the two resistances. In other words

$$\frac{Q_{eff}}{Q_0} = \frac{R_n}{R_n - R} \qquad (3)$$

This Q multiplication can be made arbitrarily large by simply letting R_n approach R.

Practical Systems

A number of systems have been used to secure this negative resistance characteristic, such as secondary emission in a tetrode (dynatron)[1] or the formation of a virtual cathode between screen and suppressor (transitron)[2]. By far the most satisfactory method to date, however, has been the use of positive feedback around a vacuum-tube amplifier.[3,4] This basic method is used in the new circuit proposed here.

Consider an amplifier of gain A and internal resistance R_i such as is represented schematically in Fig. 1A, and assume that positive feedback is introduced through the resistor R_f.

Under the assumption that the input resistance of the amplifier is so high compared to the other circuit resistances that it may be neglected—a condition easily realizable in practice—Kirchhoff's voltage law can be applied to yield the following equation

$$e_1 = i_1 R_f + i_1 R_i + A e_1 \qquad (4)$$

which can be rearranged to yield

$$Z_1 = \frac{e_1}{i_1} = -\frac{R_f}{A-1} - \frac{R_i}{A-1} \qquad (5)$$

where Z_1 is simply the effective input impedance of the circuit.

This effect is the basis for the increased selectivity of the ordinary regenerative amplifier or detector. Such a regenerative circuit, however, lacks the important characteristic of stability. Referring to Eq. 3 it is seen that appreciable multiplication of the Q is to be had only when R_n is very nearly equal to R.

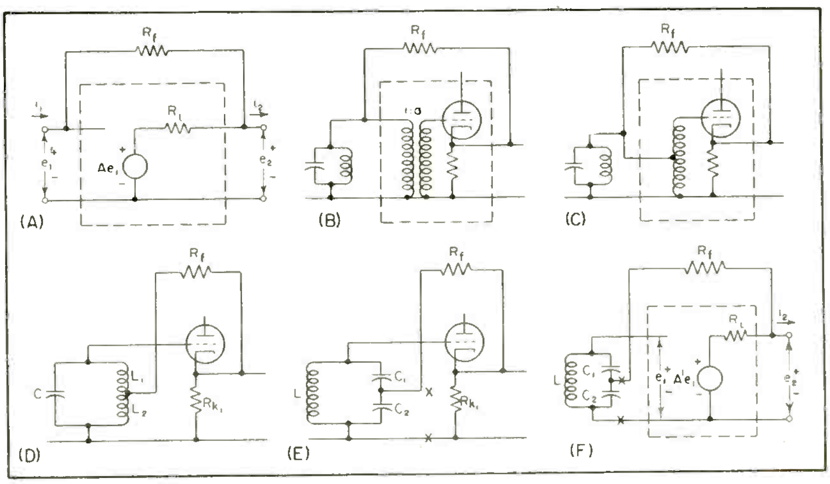

FIG. 1—Simplified circuits showing evolution of single-tube Q multiplier

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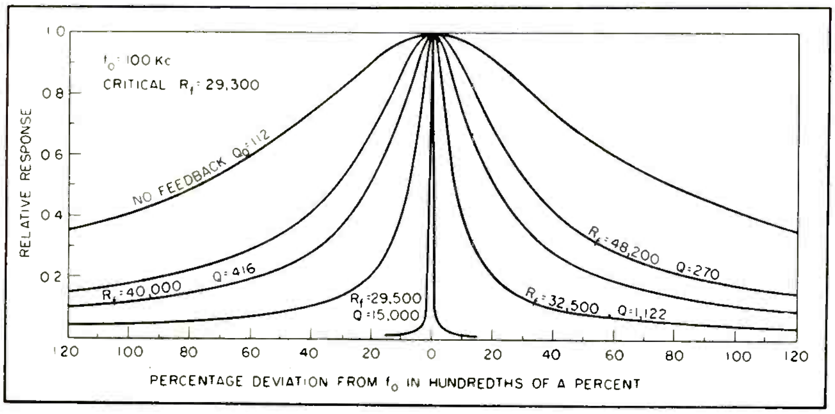

FIG. 2—Curves show selectivity obtainable with Q multiplier. Higher Q values were not tested because of measurement equipment limitations

Therefore it takes only a very slight percentage change in R_n, such as would be caused by variations in plate supply voltage, to cause the two resistances to become equal, the Q and impedance level to go to infinity, and oscillations to ensue.

One way to resolve this difficulty is to use a highly stabilized amplifier for the active element so that changes in electrode voltages and other random variations will have little effect upon the gain, and hence upon the negative resistance which is produced. Both Terman[3] and later Ginzton[4] have considered in some detail one such circuit utilizing a highly stabilized two-stage amplifier. It is the purpose of the present paper to describe a much simpler circuit which achieves essentially the same results with only a single stage. The basis of this circuit is the cathode follower. It has power gain, correct phase relation, and it has very high stability. But, it has less than unity gain. Fig. 1B shows that this drawback can be eliminated by a passive gain element, a transformer. The evolution from this circuit to the practical ones is shown in Fig. 1.

Stability Considerations

The most serious factor limiting the applicability of any positive feedback circuit is the stability. Ginzton[4] described a two-tube circuit; a more general derivation follows:

Consider the circuit shown in Fig. 1F in schematic form. Note that in the special case where $C_1 = \infty$ and $A' > 1$, this circuit reduces to the type of system considered by Gizton, while when $C_1 \neq \infty$, $A' < 1$, it represents the new circuit of Fig. 1E.

The gain of any feedback amplifier can be represented by the equation

$$A' = \frac{K}{1 - \beta K} \qquad (6)$$

In the present case, the gain of primary interest is not that of the tube itself, but rather that from points X-X (Fig. 1F) to the tube grid and through to the output at the cathode. This gain is the product of the active gain A' and what might be called the passive gain a, or gain contributed by the tapped tuned circuit

$$A = \frac{aK}{1 - \beta K} \qquad (7)$$

where

$$a = \frac{C_2 + C_1}{C_1} \qquad (8)$$

At this point the assumption is made that the output impedance of the amplifier is negligible with respect to the feedback resistor R_f. This is reasonable for an amplifier stabilized with a high degree of negative feedback. Equation 5 for the negative resistance developed across terminals X-X then reduces to

$$-R_n = -\frac{R_f}{A - 1} \qquad (9)$$

or, substituting from Eq. 7 for the actual gain of the circuit

$$-R_n = -\frac{R_f\,(1 - \beta K)}{a\,K - 1 + \beta\,K} \qquad (10)$$

Passing over the details of the derivation, it may be said that this equation is now differentiated partially with respect to the no-feedback gain K, simplified, and rearranged to yield an equation relating the fractional change in the negative resistance produced to the fractional change in the no-feedback gain. That is, an equation of the form

$$\frac{\delta R_n}{R_n} = \frac{1}{k}\;\frac{\delta K}{K} \qquad (11)$$

where the factor k might be called the stability coefficient of the system. In this case the stability coefficient is

$$k = \frac{(1 - \beta K)\,[(a + \beta)\,K - 1]}{a\,K} \qquad (12)$$

It is apparent that one would like to make the magnitude of this stability coefficient as large as possible. The maximum possible value is found by differentiating again with respect to a convenient parameter —in this case β—and setting the derivative equal to zero. The optimum value of β turns out to be

$$\beta_{opt} = -\left[\frac{a}{2} - \frac{1}{K}\right] \qquad (13)$$

The corresponding value of the stability coefficient is

$$k_{opt} = -\frac{a\,K}{4} \qquad (14)$$

It is now a simple matter to determine the optimum operating conditions for the circuit

$$\beta_{opt} = -1 \qquad (15)$$

$$k_{opt} = -K/2 \qquad (16)$$

The stability of the negative resistance is merely incidental to the matter of prime concern—the stability of the effective Q. The above results can be related to the Q stability by beginning with Eq. 3 and employing much the same process of differentiating (this time with respect to R_n) and simplifying as in Eq. 10. The result is

$$\frac{\delta Q_{eff}}{Q_{eff}} = -\left[\frac{Q_{eff}}{Q_0} - 1\right]\frac{\delta R_n}{R_n} \qquad (17)$$

Or, combining the two equations

$$\frac{\delta Q_{eff}}{Q_{eff}} = \left[\frac{Q_{eff}}{Q_0} - 1\right]\frac{2}{K}\;\frac{\delta K}{K} \qquad (18)$$

A number of interesting facts are apparent from this expression: (1) The stability is independent of the absolute value of initial Q. It is then just as easy to multiply Q from 100 to 1,000 as from 10 to 100. Thus it is important to begin with as high a Q as possible to gain maximum stability. (2) The stability is independent of the frequency, so that the circuit can be used to multiply Q anywhere in the spectrum where the stated assumptions can be met. (3) The higher the Q multiplication, the lower is the stability. For high multiplications this is approximately an inverse relation. (4) The stability increases in direct proportion to the no-feedback gain.

Another Stability Criterion

It seems logical to set up as an important design criterion of a narrow-band amplifier circuit, the amount of change in the no-feedback gain—that is, the change in the g_m with electrode voltage changes, aging, and other possible circuit variations—which can be tolerated without causing the system to break into oscillation.

As a starting point, consider again Eq. 3 for the Q multiplication, and solve this equation for R_n

$$R_n = R \frac{\frac{Q_{eff}}{Q_0}}{\frac{Q_{eff}}{Q_0} - 1} \quad (19)$$

From Eq. 3 it is apparent that the point at which oscillations begin will be that point where R_n becomes equal to R. Therefore if R is subtracted from the above expression for R_n, the result will be the absolute value of the change in R_n which can be tolerated without causing oscillations. This can then be divided by R_n, to yield the fractional change in R_n which can be tolerated

$$\left(\frac{\Delta R_n}{R_n}\right)_{tolerable} = \frac{\frac{R}{\frac{Q_{eff}}{Q_0} - 1}}{R_n} \quad (20)$$

or, making use of Eq. 19 again

$$\left(\frac{\Delta R_n}{R_n}\right)_{tolerable} = \frac{Q_0}{Q_{eff}} \quad (21)$$

But here again the negative resistance is merely a derived characteristic of the circuit. What is really wanted is the permissible change in the no-feedback gain K. It is apparent from the optimum operating conditions which were derived, and from Eq. 10 that the negative resistance can be represented as

$$R_n = R_f \left(\frac{K+1}{K-1}\right) \quad (22)$$

Then if R_{n1} represents the value of the negative resistance at some particular chosen operating point of the circuit and R_{nc} the critical value of negative resistance at which oscillations occur, Eq. 21 above can be rewritten

$$\left(\frac{\Delta R_n}{R_n}\right)_{tolerable} = \frac{R_{n1} - R_{nc}}{R_{n1}} = \frac{Q_0}{Q_{eff}} \quad (23)$$

Substituting from Eq. 22 and simplifying, this becomes

$$\frac{R_{n1} - R_{nc}}{R_{n1}} = \frac{2(K_c - K_1)}{K_1 K_c - K_1 + K_c - 1} \quad (24)$$

where K_1 is the no-feedback gain at the particular operating point chosen above, and K_c is the no-feedback gain at the critical point.

Now let $K_c = K_1 + \Delta K$. Then

$$\frac{R_{n1} - R_{nc}}{R_{n1}} = \frac{2\Delta K}{K_1^2 + K_1 \Delta K + \Delta K - 1} \quad (25)$$

which, by Eq. 21 is equal to $\frac{Q_0}{Q_{eff}}$. Equating and solving for ΔK gives:

$$\Delta K = \frac{K_1^2 - 1}{\frac{2 Q_{eff}}{Q_0} - (K_1 + 1)} \quad (26)$$

If this equation is now divided by K_1 the result is the fractional change in no-feedback gain K ($=g_m R_k$) which can be tolerated without oscillations

$$\left(\frac{\Delta K}{K}\right)_{tol} = \frac{1 - \frac{1}{K^2}}{\frac{1}{K}\left(2\frac{Q_{eff}}{Q_0} - 1\right) - 1} \quad (27)$$

In practice, K is almost always kept much larger than 1, and the Q multiplication much larger than a half, so that a somewhat simpler working formula may be obtained:

$$\left(\frac{\Delta K}{K}\right)_{tol} \approx \frac{1}{\frac{\frac{Q_{eff}}{Q_0}}{\frac{K}{2}} - 1} \quad (28)$$

This is a most interesting expression. For suppose that at some particular operating point a

$$\left.\frac{Q_{eff}}{Q_0}\right|_a = \left.\frac{K}{2}\right|_a \quad (29)$$

The above equation then goes to infinity, signifying an infinite change in K necessary to cause oscillation. Further, suppose that

$$\left.\frac{Q_{eff}}{Q_0}\right|_a < \left.\frac{K}{2}\right|_a \quad (30)$$

Then the fractional change in K necessary to cause oscillations is a negative number greater than 1. But this would require a negative gain, which, of course, is impossible in a vacuum-tube amplifier. It can be concluded, therefore, that if at any operating point, the condition

$$\left.\frac{Q_{eff}}{Q_0}\right|_a \leqq \left.\frac{K}{2}\right|_a \quad (31)$$

is met, or, in other words if the circuit constants are adjusted so that the no-feedback gain K ($=g_m R_k$) is always greater than twice the degree of Q multiplication which is desired, there will be no chance of the circuit breaking into oscillation no matter how much the g_m of the tube may change with aging, changes in electrode voltages, shock and so on.

Here, then, is the fundamental contribution of this new circuit. Without any substantial increase in the complexity over the ordinary regenerative system, it has made possible attainment of arbitrarily high Q multiplications, while at the same time retaining the absolute stability of the ordinary amplifier.

Practical Circuit

It is not possible to set down any hard and fast rule as to the magnitude of the $g_m R_k$ product which may be obtained. Experience has shown, however, that with a 6AK5 and a supply voltage of 200 volts, values of about 100 are easily attainable. With higher supply voltages, correspondingly higher values of the $g_m R_k$ product may be realized.

Now suppose that a relatively modest degree of Q multiplication—say 10—is all that is wanted. (This still will allow Q's of the order of 2,000 to 3,000 if a good coil is used). The above equations then become

$$\left(\begin{matrix}\text{\% change in} \\ Q_{eff}\end{matrix}\right) = \frac{1}{5.5}\left(\begin{matrix}\text{\% change in} \\ g_m\end{matrix}\right) \quad (32)$$

Oscillation impossible

In other words, the percentage change in the Q is only approximately a sixth of the percentage change in the g_m which caused it, and it will be impossible to cause the circuit to oscillate no matter how much the g_m may change with shifts in plate voltage and other circuit parameters.

Even for the relatively high Q multiplication of 100, which would

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correspond to possible Q's of the order of 30,000, stability is excellent.

$$(\% \text{ change in } Q_{eff}) = 2\,(\% \text{ change in } g_m) \quad (33)$$

$$100\% \text{ change in } g_m \text{ to cause oscillation}$$

This is still well within the practical range of operation, if a power supply of any reasonable regulation is used.

Experimental Results

The curves of Fig. 2 show the results obtainable from a typical circuit of this new type. These curves were taken by applying a variable-frequency, constant-voltage signal to the Q multiplier circuit through an isolating stage and measuring the output voltage as a function of the frequency. The no-feedback curve is a plot of output voltage versus frequency when the feedback circuit was opened, or when $R_f = \infty$ and the circuit was operating as an ordinary cathode follower. The other curves show the effect of reducing the feedback resistor closer and closer to the critical value of 29,300 ohms. The maximum Q value of 15,000 shown was by no means the limit obtainable with the circuit. There was simply no measuring equipment available precise enough to allow a reliable set of data to be taken for higher Q's.

Theory indicates that the shape of the response curve should be unaltered by the Q multiplication. This was checked by plotting data taken for several values of multiplication on the same graph as the universal resonance curve. In every case the results were identical. This means that these circuits may profitably be cascaded or staggered, using the identical means of calculation as for ordinary resonant circuits.

Experiments have verified the two stability relations. In both cases, the stability turned out to be slightly higher than predicted.

Practical Suggestions

For the convenience of the designer, it might be well here to summarize a few practical hints which have been discovered in the course of working with this circuit. First of all, for reference, the actual circuit used for the preceding experimental measurements is reproduced in Fig. 3A.

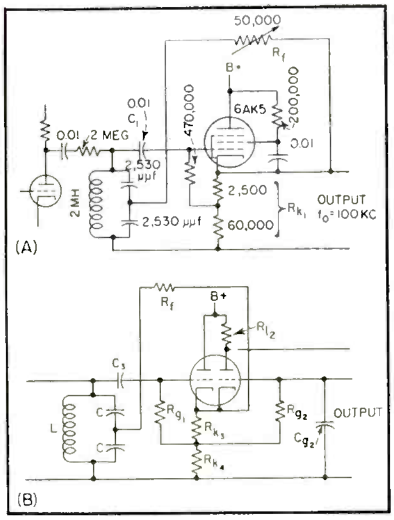

FIG. 3—Practical circuits of single-tube Q multiplier. In *B* the first triode section serves as the Q multiplier and the second as a grounded-grid amplifier

The exact critical value of the resistance R_f is easily found from Eq. 3. Remembering that the negative resistance is developed across only half of the tuned circuit in this system, Eq. 3 actually becomes:

$$\frac{Q_{eff}}{Q_0} = \frac{R_n}{R_n - R/4} \quad (34)$$

But R_n is given by Eq. 22. Substituting and regrouping gives

$$\frac{Q_{eff}}{Q_0} = \frac{1}{1 - \dfrac{R/4}{R_f}\,\dfrac{(K-1)}{(K+1)}} \quad (35)$$

from which it is apparent that the critical resistance is

$$R_f = \frac{R}{4}\,\frac{(K-1)}{(K+1)} \quad (36)$$

For design purposes this value can be taken simply as one-fourth of the tuned circuit impedance. If the circuit is operating properly, oscillations will ensue for all values of R_f less than this value. For $R_f = \infty$ the circuit operates as a cathode follower, and as R_f decreases toward the critical value, the Q multiplication increases without limit.

The actual R_k to be used in computing the $g_m R_k$ product is the cathode resistor R_{k1} in parallel with the series combination of R_f and one-fourth of the impedance of the tuned circuit—that is, approximately the cathode resistor in parallel with one-half the tuned circuit impedance in the multiplier.

The grid biasing connection shown in Fig. 3A is used for the purpose of increasing the $g_m R_k$ product, and hence the stability. Using this arrangement, a large cathode resistor can be used without increasing the grid bias excessively and thus reducing g_m.

Somewhat higher stabilities are obtained by using pentode as in Fig. 3A, instead of the triode discussed previously. The screen should be by-passed to the cathode. Otherwise the tube will operate as a triode. If only moderate multiplications are needed, however, the double triode circuit shown in Fig. 3B may be found useful. Here the first section is used as a Q multiplier, and the second as a grounded-grid amplifier.

The source impedance should be kept high, either by the use of a series resistor as in Fig. 3A, or by designing the preceding stage for a high output impedance. If high Q multiplications are sought, the series resistor is preferable, in conjunction with a low output impedance for the previous stage.

The phase shift must be kept to a minimum to avoid frequency shift as the Q multiplication is changed.

When the split inductor variation of Fig. 1D is used, the cathode resistor R_{k1} may be omitted. This allows about a 2-to-1 increase in stability.

The signal input should be kept relatively low for best results. Experience with the type 6AK5 has shown that inputs much more than a volt or two result in reduced effective Q multiplication due to curvature of the tube characteristic.

It is possible to raise the Q of a coil alone by use of the circuit in Fig. 1D with the capacitor omitted. Use in such a manner suggests a number of additional applications for the circuit.

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